

Jožef Stefan Institute



**UNIVERSITY OF
CAMBRIDGE**

Modelling the collapse of complex societies

Sabin Roman

Work on collapse



My articles on societal dynamics and collapse.

My field: Societal collapse and cliodynamics



Clio, the muse of history (Berlin)

- Interest in topic since late 1700s (Gibbon and Malthus)

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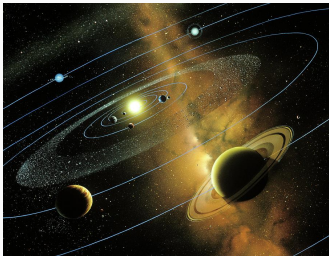


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- Interest in topic since late 1700s (Gibbon and Malthus)
- Much written, mostly qualitative insights
- Real progress?
- Mathematical modelling is rare
- Data scarce and sparse
- Apparent intractability

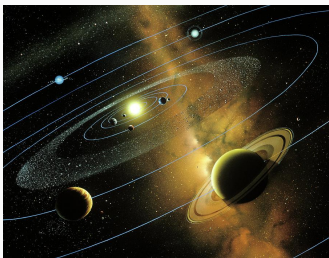
The first scientific success

The first scientific success



Kepler's laws of planetary motion, published in *Astronomia Nova*

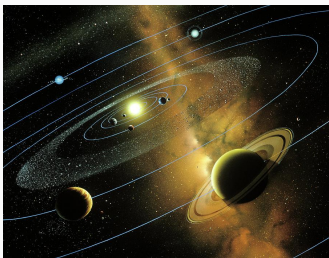
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Gravity is the dominant force over large spatial scales (solar system).

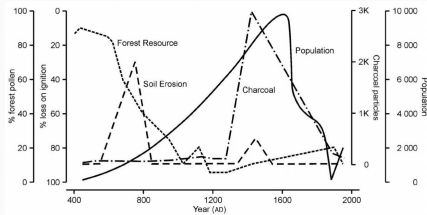
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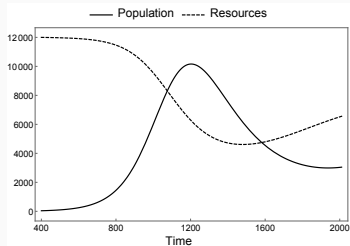
Kepler's laws of planetary motion, published in *Astronomia Nova*

Gravity is the dominant force over large spatial scales (solar system).
What is the equivalent of “gravity” in terms of social forces that persist over long temporal scales?

The first model in the literature



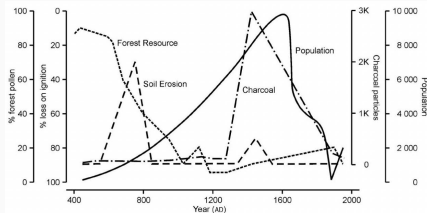
Archaeological record



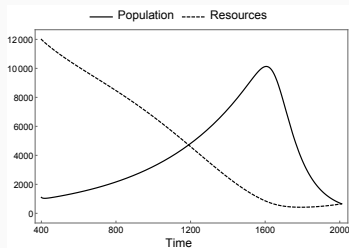
Brander and Taylor (1998) results

Economic model gives a poor fit.

Easter Island improvement



Archaeological record



Roman et al. (2017) results

Ecological model with same parametrization as economic model.

Interpretation

- x is the population
- y is the resources, e.g., trees, fish
- z is the wealth e.g. food stocks, housing, tools

Equations

$$\dot{x} = \left(b - de^{-z/(\rho x)} \right) x$$

$$\dot{y} = ry \left(1 - \frac{y}{K} \right) - \alpha xy$$

$$\dot{z} = \alpha xy - sx \left(1 - e^{-z/(\rho x)} \right) - cz$$

Roman, S., Bullock, S., & Brede, M. (2017). Coupled societies are more robust against collapse: A hypothetical look at Easter Island. *Ecological Economics*, 132, 264-278.

Modelling criteria

- Historical fit
- Plausibility of underlying assumptions

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- Robustness
- Diversity / flexibility
- Reproducible, ease of communication

Maya Civilisation model

The dynamical model we propose is given by

$$\dot{x}_s = [(1 - \tau) + \tau p_s] \beta n x - \beta n^{-\delta} x_s + \sigma [(1 - \theta(n)) x_b - \theta(n) x_s]$$

$$\dot{x}_i = \tau p_i \beta n x - \beta n^{-\delta} x_i$$

$$\dot{x}_b = -\beta n^{-\delta} x_b - \sigma [(1 - \theta(n)) x_b - \theta(n) x_s]$$

$$\dot{y} = w_t r y (1 - y / (w_t K)) - s d n x$$

$$\dot{z} = b x_b - m z$$

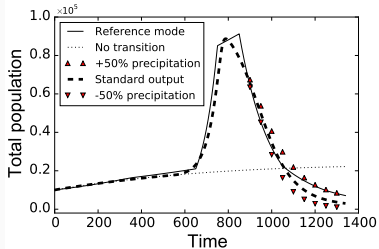
where x_s, x_i, x_b are specialisations, y resources, z infrastructure, and x, n , and $\theta(n)$ are given by:

$$x = x_s + x_i + x_b \quad n = w_t \frac{x_s + \alpha x_i}{x_s + x_i + x_b} (1 - e^{-y / (w_t c K)})$$

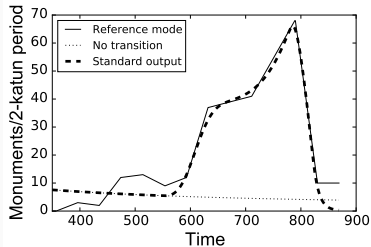
$$\theta(n) = \frac{1}{1 + e^{-2k(n - n_b)}}$$

Roman, S., Palmer, E., & Brede, M. (2018). The dynamics of human–environment interactions in the collapse of the Classic Maya. *Ecological Economics*, 146, 312-324.

Maya civilisation results



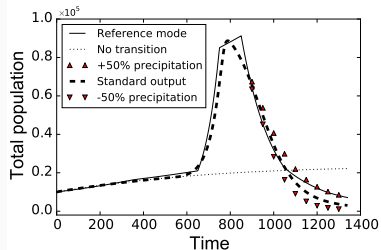
Population fit



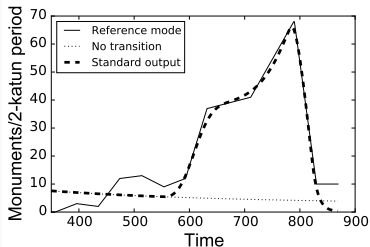
Monuments fit

Lowland Maya results (Roman et al., 2018)

Maya civilisation results



Population fit



Monuments fit

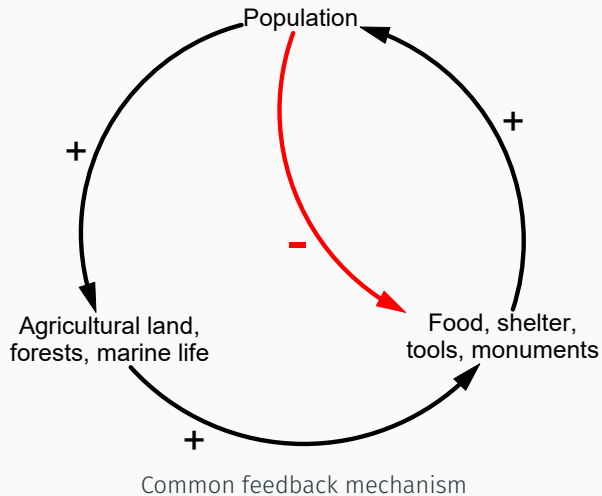
Lowland Maya results (Roman et al., 2018)

Joseph Tainter, world leading expert on collapse:

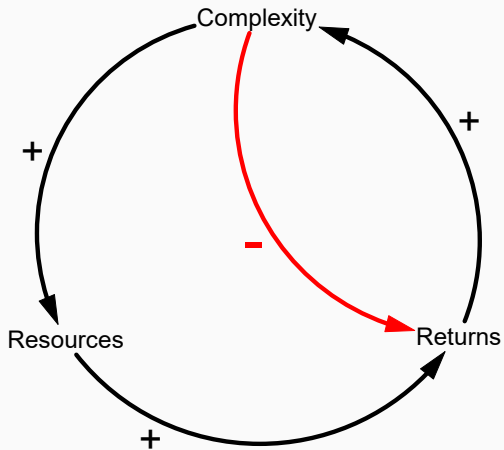
We need more model building of this calibre.

Tainter, Joseph. "Modelling the mysterious Maya." *Nature Sustainability* 1.2 (2018): 79-80.

The feedback mechanisms



General mechanism



Diminishing returns to investment in complexity

Another instance: The Roman Empire

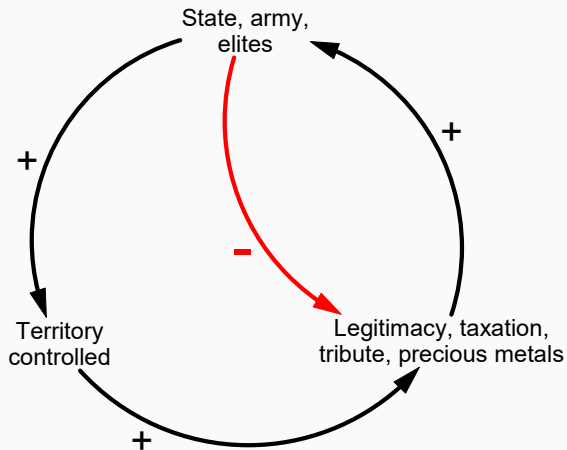
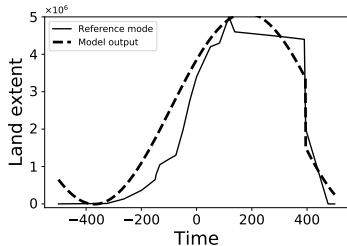
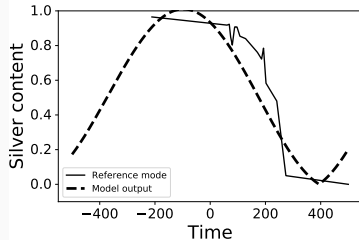


Diagram for the Roman Empire

Fitting the historical record



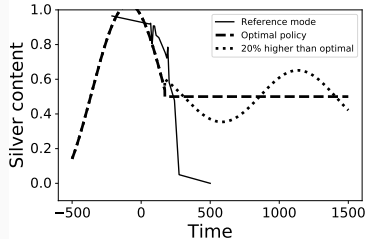
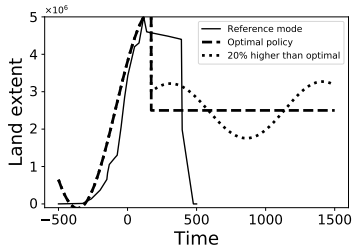
Territorial evolution



Coin debasement

Results from Roman and Palmer (2019).

Saving the empire



Cutting army, territory and fixing coin purity.

Separation of time scales

Dynamics	Time scale (years)
Biological evolution, tool making, climate effects (e.g., ice ages)	100,000

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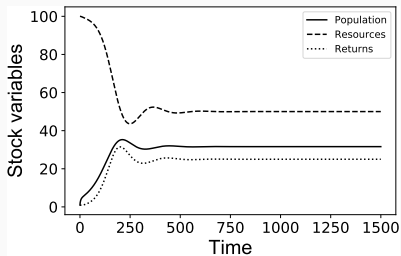
$$\dot{x} = b\frac{z}{x} - dx$$

$$\dot{y} = ry\left(1 - \frac{y}{K}\right) - \alpha x^2 y$$

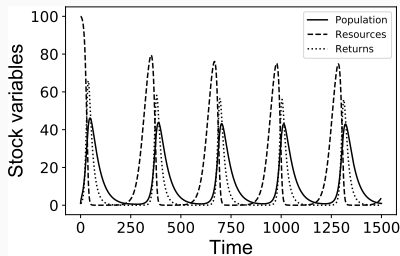
$$\dot{z} = \alpha x^2 y - cz$$

Model for population, resources and returns.

Model dynamics



Stable fixed point



Limit cycle

Critical parameter value given by:

$$\alpha_c = \frac{2cd + (c + 2d)^2}{4Kb} \left(1 + \sqrt{1 + \frac{8rcd(c + 2d)}{[2cd + (c + 2d)^2]^2}} \right)$$

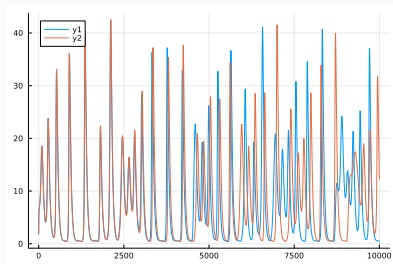
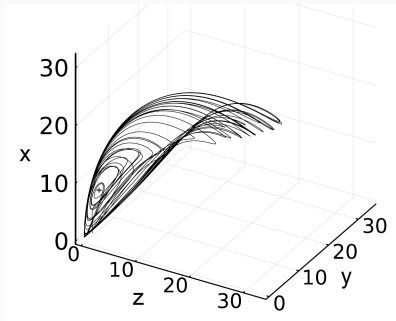
$$\dot{x}_i = b \frac{z_i}{x_i} - dx_i$$

$$\dot{y}_i = ry_i \left(1 - \frac{y_i}{K}\right) - \alpha_i x_i y_i \sum_{j=1}^N x_j A_{ij}$$

$$\dot{z}_i = \alpha_i x_i y_i \sum_{j=1}^N x_j A_{ij} - cz_i$$

Interaction between regions is given by adjacency matrix A .

Chaos and synchronization



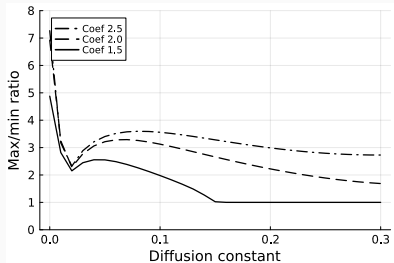
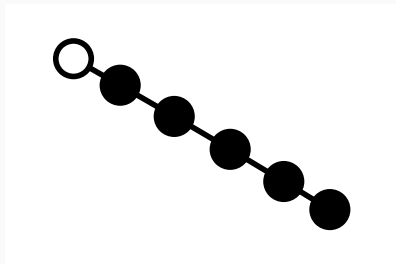
$$A = \begin{bmatrix} 1 + \delta & 1 & 1 \\ 1 & 1 + \delta & 1 \\ 1 & 1 & 1 + \delta \end{bmatrix}$$

Dynamics is chaotic and synchronized on a 3-dimensional manifold. Lyapunov exponent $\simeq 3.6 \times 10^{-3}$ (close to upper limit). Inverse is $\simeq 280$ years, compatible with lifespan of empires.

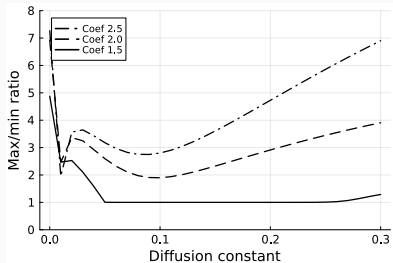
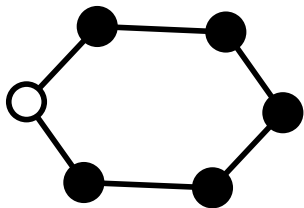
$$\begin{aligned}\dot{x}_i &= b \frac{z_i}{x_i} - dx_i - \sigma \sum_{j=1}^N x_j B_{ij} \\ \dot{y}_i &= ry_i \left(1 - \frac{y_i}{K}\right) - \alpha_i x_i^2 y_i \\ \dot{z}_i &= \alpha_i x_i^2 y_i - cz_i\end{aligned}$$

Where σ is the diffusion constant and B is the network Laplacian.

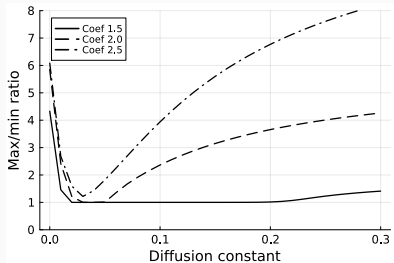
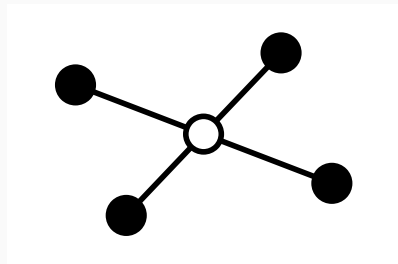
Inducing sustainability I



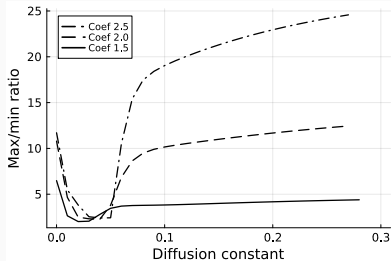
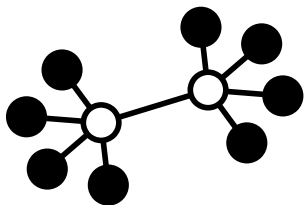
Inducing sustainability II



Inducing sustainability III



Inducing sustainability IV



Dynamic and game theoretic modelling of societal growth, structure and collapse

Roman, Sabin (2018) Dynamic and game theoretic modelling of societal growth, structure and collapse. University of Southampton, Doctoral Thesis, 136pp.

Recent type: [Thesis \(Doctoral\)](#)

Abstract

The dynamics and structure of societies have long been a puzzle to archaeologists, historians and social scientists in general. In particular, increases in social inequality and the possibility of societal collapse are two deeply distressing prospects for any society. In this thesis I provide two contributions to the literature of societal collapse and the possibility of the emergence of social inequality.



PLOS ONE

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RESEARCH ARTICLE

Global history, the emergence of chaos and inducing sustainability in networks of socio-ecological systems

Sabin Roman Francesco Berardi

Published: November 15, 2023 • <https://doi.org/10.1371/journal.pone.0280391>

Article	Authors	Metrics	Comments	Media Coverage	Peer Review
1					

Abstract

Introduction
Background
Model specification

Abstract

In this study, we propose a simplified model of a socio-environmental system that accounts for population, resources, and wealth, with a special focus on population contributions to the resource extraction term. Given its structure, an analytical treatment of attractors and bifurcations is possible. In particular, a Hopf bifurcation from a stable fixed point to a limit cycle emerges.

Long-term Feedback Mechanisms Underlying Societal Growth and Collapse

26 Pages • Posted: 25 Oct 2023 • Last revised: 27 Nov 2023

Sabin Roman
Center for the Study of Existential Risk

Date Written: October 2, 2023

Abstract

In this work we address some common methodological pitfalls in understanding societal collapse and propose a framework to remediate them. With this goal we reformulate and extend Tainter's theory of societal



2. Theories and Models: Understanding and Predicting Societal Collapse

Sabin Roman

Chapter of: [The Era of Global Risk: An Introduction to Existential Risk Studies](#) (pp. 27–54)

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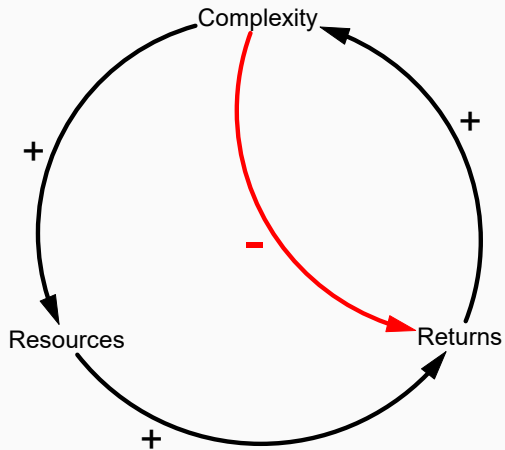
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Why do societies collapse? This chapter offers approaches to understanding which quantitative and qualitative models of societal dynamics can predict societal collapse and which exogenous and endogenous factors contribute to it.

The big picture papers.

Thank you!

Main lesson 1: Qualitative



Diminishing returns to investment in complexity

Building a modelling dictionary:

- $x(1 - x)$ logistic growth
- xy mass action term
- $b - de^{-z/(\rho x)}$ effect of wealth on birth rates
- $w_tsd(x_s + \alpha x_i) \exp\left(\frac{-y}{w_tCK}\right)$ agricultural production
- $\sigma[(1 - \theta(n))x_b - \theta(n)x_s]$ changing between specializations
- $\frac{z_1}{p_1z_2} - 1$ effect of debasement on army and land

Inequality, migration, natality, agricultural practices, the military doctrine, religion, trade, bureaucracy, natural disasters, resource depletion, political incentives, biological predispositions, economic policy, relations with neighbours, ruler's character ...

Main lesson 3: Quintessential

Inequality, migration, natality, agricultural practices, the military doctrine, religion, trade, bureaucracy, natural disasters, resource depletion, political incentives, biological predispositions, economic policy, relations with neighbours, ruler's character ...

For N concepts we have $N!$ permutations of their importance (aka theories). $N = 14$ we get 87 billion possible theories. Which one is "right"?

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Time scale separation helps.