Detectability of noisy signals for photothermal and photoacoustic reconstruction

Burgholzer P(1)*, Mayr G(2), Thummerer G(2), Haltmeier M(3)

(1) RECENDT, Research Center for Non-Destructive Testing, Linz, Austria
(2) Josef Ressel Centre for Thermal NDE of Composites, University of Applied Sciences Upper Austria, Wels, Austria
(3) Department of Mathematics, University of Innsbruck, Innsbruck, Austria

*Corresponding author's email: peter.burgholzer@recendt.at

Background – in this work, we combine the different scientific fields of information theory, thermodynamics, regularization theory and non-destructive imaging, especially for photoacoustic and photothermal imaging [1]. The goal is to get a better understanding of how information gaining for subsurface imaging works and how the spatial resolution limit can be overcome by using additional information. Here, the resolution limit in photoacoustic and photothermal imaging is derived from the irreversibility of attenuation of the pressure wave and of heat diffusion during propagation of the signals from the imaged subsurface structures to the sample surface, respectively. The acoustic or temperature signals are converted into so-called virtual waves, which are their reversible counterparts and which can be used for image reconstruction by well-known ultrasound reconstruction methods. The conversion into virtual waves is an ill-posed inverse problem which needs regularization. The reason for that is the information loss during signal propagation to the sample surface, which turns out to be equal to the entropy production. As the entropy production from acoustic attenuation is usually small compared to the entropy production from heat diffusion, the spatial resolution in acoustic imaging is higher than in thermal imaging. Therefore, it is especially challenging to overcome this resolution limit for thermographic imaging. Incorporating additional information such as sparsity and non-negativity in iterative regularization methods gives a significant resolution enhancement, which is experimentally demonstrated by one-dimensional imaging of thin layers with varying depth or by three-dimensional imaging, either from a single detection plane or from three perpendicular detection planes on the surface of a sample cube (Fig. 1).

The virtual wave concept – the formal relationship between temperature field $T(\mathbf{r},t)$ and virtual wave field $T_{\mathrm{virt}}(\mathbf{r},t')$, for the same position vector $\mathbf{r}$ but different time scales t and t', is given by a Fredholm integral of the first kind:

$$T(\mathbf{r},t) = \int_{-\infty}^{\infty} K(t,t')\, T_{\mathrm{virt}}(\mathbf{r},t')\, \mathrm{d}t' \quad \text{with} \quad K(t,t') = \frac{c}{\sqrt{\pi\alpha t}} e^{-\frac{c^2 (t')^2}{4\alpha t}} \quad \text{for} \quad t > 0. \qquad \text{Eqn. 1}$$

The thermal diffusivity α and the virtual speed of sound c are the characteristic parameters for heat and virtual wave propagation. While $T(\mathbf{r},t)$ obeys the heat equation, $T_{\mathrm{virt}}(\mathbf{r},t')$ fulfils the photoacoustic wave equation. The above equation is valid for a Dirac-Delta-like heating in time domain, for other temporal heating functions the solution is the convolution in time with this function. The kernel $K(t,t')$ works as transition function and contains the characteristic parameters α and c. It is important to note that $K(t,t')$ is independent of the position vector $\mathbf{r}$.

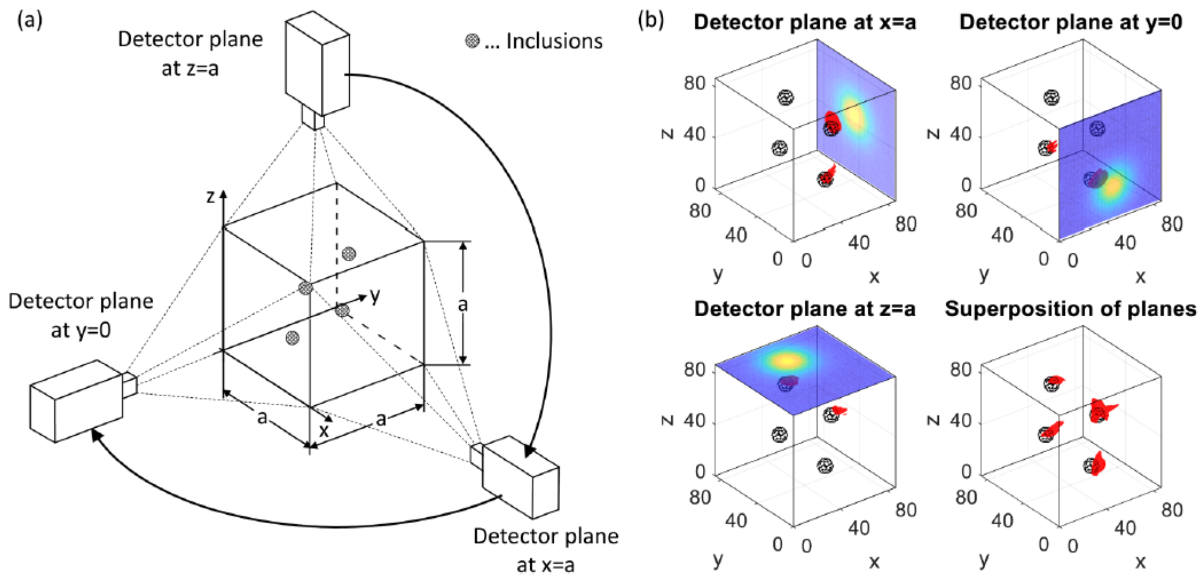

Fig. 1. (a) Experimental setup and detection planes of the thermographic measurements and **(b)** the isosurface illustration of the reconstructed internal heat sources obtained with reconstruction using sparsity and positivity for three different detector planes and a superposition of the single detector plane reconstructions [1]. The steel spheres had depths of 4.3 mm, 7.5 mm and 10.7 mm from the detection planes.

That means, when considering surface temperature data, that is acquired by an IR-camera, we have a pixel-wise transformation. Since thermography data is discrete in time and space, we have to discretize the above relationship between $T(\mathbf{r}, t)$ and $T_{\text{virt}}(\mathbf{r}, t')$:

$$\mathbf{T} = \mathbf{K}\mathbf{T}_{\text{virt}} \qquad \text{Eqn. 2}$$

The matrix $\mathbf{K}$ has rapidly decaying singular values, hence calculating $\mathbf{T}_{\text{virt}}$ based on $\mathbf{T}$ is a severely ill-posed inverse problem. Due to this, we need regularization. In principle, we can employ direct or iterative regularization methods. By using the truncated singular value decomposition (T-SVD) or Tikhonov regularization the pseudo-inverse or Moore-Penrose $\mathbf{K}^{\dagger}$is approximated.

In Figure 2, the process steps of the virtual wave concept based on simulated data are depicted. Here we consider an experiment, where a laser pulse heats a 2D sample to be imaged. Because of the transparency of the bulk material in the wavelength of the laser only the inclusions are heated. Consequently, they act as internal heat sources and we have a certain initial temperature distribution $T_0(\mathbf{r})$. The heat diffuses to the surface, where the spatial and temporal surface temperature signal $T(\mathbf{r}, t)$ is recorded. Then, as mentioned before, we have to solve a severely ill-posed inverse problem. In this case, for the computation of the virtual wave field $T_{\text{virt}}(\mathbf{r}, t)$, we have employed ADMM, which is an efficient iterative algorithm for constrained optimization. We have incorporated positivity and sparsity as prior information. Sparsity is respected because we assume that we only have a few point scatterers in the sample under test and hence a sparse virtual wave field. As one can see the computed virtual wave field $T_{\text{virt}}(\mathbf{r}, t)$ meets the ideal virtual wave field $T_{\text{virt}}^{\text{ideal}}(\mathbf{r}, t)$ well but the blurring due to information loss cannot be fully eliminated. As a last step, we can apply well developed ultrasonic methods in order to reconstruct the initial temperature distribution $T_0^{\text{rec}}(\mathbf{r})$. Despite the rather strong blurring of $T_{\text{virt}}(\mathbf{r}, t)$for deeper lying structures, even deeper structures can be well reconstructed by ultrasonic reconstruction techniques because they “average” signals from all directions and therefore the noise is reduced very effectively in $T_0^{\text{rec}}(\mathbf{r})$.

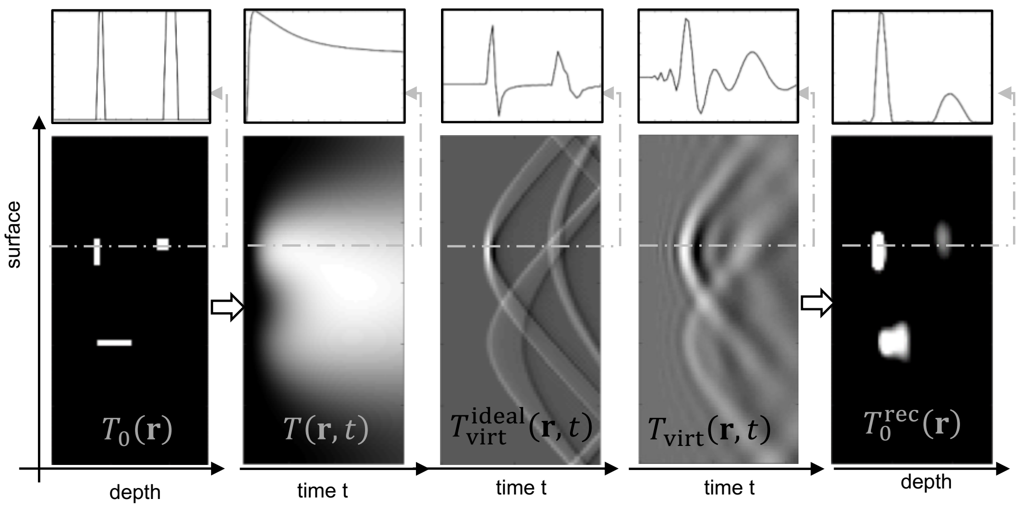

Fig. 2. Process steps of the virtual wave concept: First, the internal sources are heated, by e.g. optical excitation, yielding a certain initial temperature distribution $T_0(\mathbf{r})$. Simultaneously, the surface temperature signal $T(\mathbf{r},t)$ is measured using an IR-camera. Then the temperature signal is transformed into an acoustic virtual wave signal $T_{\mathrm{virt}}(\mathbf{r},t)$. When comparing $T_{\mathrm{virt}}(\mathbf{r},t)$ with the ideal virtual wave field $T_{\mathrm{virt}}^{\mathrm{ideal}}(\mathbf{r},t)$, we see that the former is blurred because of entropy production during heat diffusion. As a final step, we can employ ultrasonic methods to reconstruct the initial temperature distribution $T_0^{\mathrm{rec}}(\mathbf{r})$.

[1] P. Burgholzer, G. Mayr, G. Thummerer, M. Haltmeier, Linking information theory and thermodynamics to spatial resolution in photothermal and photoacoustic imaging, J. Appl. Phys. 128 (2020) 171102 https://doi.org/10.1063/5.0023986.