Photoacoustic reconstruction formulas exploiting known location of 2D initial pressure

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Background – The image reconstruction problem in photoacoustic tomography (PAT) is inherently three-dimensional because of the propagation of waves in 3D space. However, with an advanced measurement technique based on integrating line detectors an analogous reconstruction problem also occurs in two dimensions [1]. The 2D wave equation is different from the 3D wave equation, because the pressure in 2D does not become zero after some finite time. This phenomenon is also reflected in known inversion formulas which use all pressure signals up to infinite time. This leads to several practical disadvantages due to noise, reflection artifacts and damping. Therefore, in practice, the inversion formulas are cut off at a certain time, leading to errors in the reconstruction. In 3D, this is not the case, because in contrast it is necessary to integrate only over a finite time interval, which is derived from the known location of the object (Huygens' principle). In this work, we also establish similar formulas in 2D using pressure data only from a finite time interval.

Methods – In the following, we consider a domain $\Omega \subset \mathbb{R}^2$ in the plane with a smooth boundary $\partial\Omega$. Moreover, we suppose that the initial pressure $f: \mathbb{R}^2 \to \mathbb{R}$ of the two-dimensional wave equation

$$\begin{aligned} \partial_t^2 p(x,t) - c^{-2}\Delta p(x,t) &= 0, \\ p(x,0) &= f(x), \\ \partial_t p(x,0) &= 0 \end{aligned}$$

is located in some smaller domain $\Omega' \subset \Omega$, where $(x,t) \in \mathbb{R}^2 \times (0,\infty)$, ∂_t is the time derivative, Δ denotes the Laplacian with respect to the space variable and c describes the speed of sound in a medium. As mentioned before, we are interested to recover the initial pressure f from photoacoustic pressure signals that are measured only within a certain finite time interval $[T_1(y), T_2(y)]$ for each detector located at $y \in \partial\Omega$. As we will see in the results, theoretical exact reconstruction of f is possible by explicit inversion formulas if the data of the pressure wave is collected between the times points

$$0 \le T_1(y) \le c^{-1} \inf\{\|y-z\| \mid z \in \Omega'\}, \; T_2(y) \ge c^{-1} \sup\{\|y-z\| \mid z \in \Omega'\}. \qquad \text{Eqn. 1}$$

Fig. 1. and 2. visualize the upper bound of $T_1(y)$ and the lower bound of $T_2(y)$ in Eqn. 1 with a simple initial pressure and the corresponding pressure signals. It should be noted that even if an optimal domain Ω' is not known, one can at least take $\Omega' = \Omega$, since the initial pressure is contained in Ω. Such a situation has previously been studied in [2]. Opposed to [2], previous formulas, as for example derived in [3,4], require pressure signals on the whole time interval $(0,\infty)$.

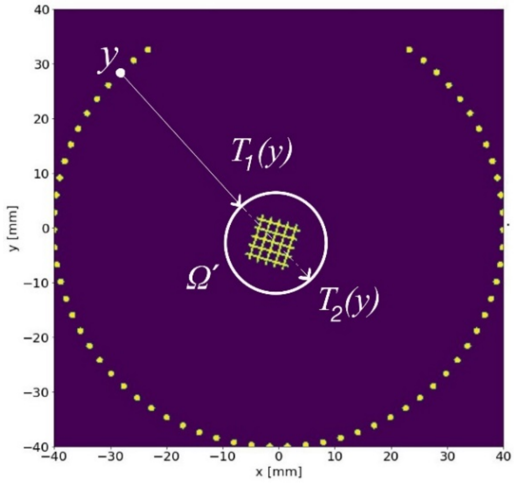

Fig. 1. A simple initial pressure located in the domain Ω'. The yellow dots describe the location of the detector points on the boundary of Ω.

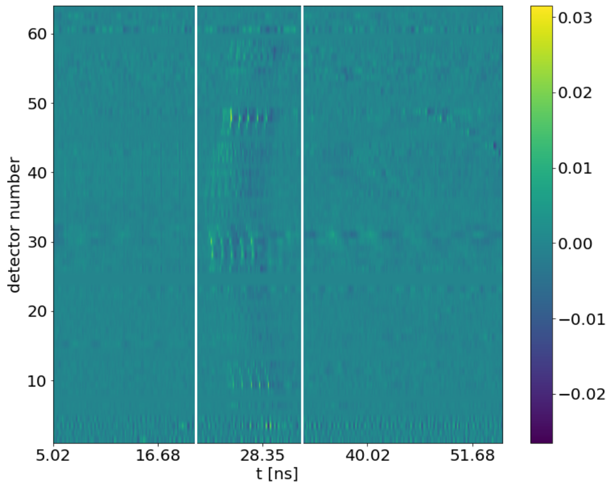

Fig. 2. Corresponding measured acoustic signal. The area between the two white stripes describes the data between the constant time points $T_1(y)$ and $T_2(y)$.

Results – The following main theoretical result of this work gives an inversion formula for 2D PAT using measured pressure signals on finite time intervals only. It is an extension of the results in [2], where $T_1(y)$ equals zero and $T_2(y)$ is set to the diameter of domain Ω for all $y \in \partial\Omega$.

Theorem. Let $\Omega \subset \mathbb{R}^2$ be bounded by an ellipse $\partial\Omega = Q\{x \in \mathbb{R}^2 \mid (x_1/a)^2 + (x_2/b)^2 \leq 1\}$, where $a, b > 0$, Q is an orthogonal matrix, $\Omega' \subset \Omega$ and $T_1(y)$ and $T_2(y)$ are such that Eqn. 1 for all $y \in \partial\Omega$ is satisfied. Furthermore, suppose that initial pressure $f\colon \mathbb{R}^2 \to \mathbb{R}$ vanishes outside of Ω'. Then, the following reconstruction formula for all $x \in \Omega$ holds:

$$f(x) = \frac{1}{\pi} \nabla_x \cdot \int_{\partial\Omega} \nu(y) \int_{T_1(y)}^{T_2(y)} k_{T_2(y)}(\|x - y\|, t)\, p(y,t)\, dt\, d\sigma(y).$$

Here, ∇_x denotes the gradient with respect to x and $\nu(y)$ the outward unit normal vector of $\partial\Omega$ at y. For the definition of the function $k_{T_2(y)}\colon (0, T_2(y)) \times \big(0, T_2(y)\big) \to \mathbb{R}$ we refer to [2] where the same kernel function has been used.

Conclusion – In this work, we derived a new inversion formula on finite time intervals in 2D for recovering the initial data in PAT that make use of additional available information of the location of the acoustic pressure source. In the final talk, we will also present reconstruction results from simulated as well as experimental data, demonstrate benefits of the new formula and compare the results with previous formulas requiring data for all times.

References

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