



# Simultaneous reconstruction of density and thermal conductivity depth profiles in sintered metal powder compacts using a novel inverse thermal-wave method

Kooshki S<sup>(1)</sup>, Melnikov A<sup>(1)</sup>, Mandelis A<sup>(1)\*</sup>

(1) Center for Advanced Diffusion-Wave and Photoacoustic Technologies (CADIPT), Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario M5S 3G8, Canada

(2) Institute for Advanced Non-Destructive and Non-Invasive Diagnostic Technologies (IANDIT), University of Toronto, Ontario M5S 3G8, Canada

\*Corresponding author's email: [mandelis@mie.utoronto.ca](mailto:mandelis@mie.utoronto.ca)

An inverse method is proposed to determine simultaneous density and thermal conductivity depth profiles in a general inhomogeneous solid. A novel integral-equation based solution for thermal-wave boundary value problems is introduced combined with a 2-stage iterative inverse algorithm to reconstruct both thermal conductivity and density depth profiles as expansions of judiciously selected continuous depth functions with several unknown parameters.

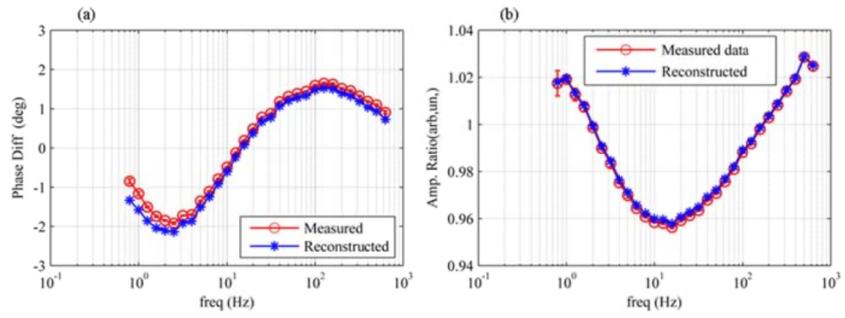
**Introduction** – Historically, many research reports have been presented for the reconstruction of thermophysical properties of inhomogeneous materials using inverse algorithms and numerical methods [1-4]. Unlike the foregoing single property reconstructions to-date, the novelty of the current work is in the simultaneous reconstruction of *both* density and thermal conductivity from the measurement of amplitude and phase of the modulated surface temperature vs. frequency.

**Methodology** – The proposed inverse method which is based on photothermal frequency scans of depth inhomogeneous metal powder compacts used in automotive manufacturing, was coupled with the depth-variable thermal-wave field model and applied in two stages. In the first stage, an initial guess for the thermal conductivity depth profile was assumed and the density depth profile was expanded in a bounded series of depth-continuous basis functions with several unknown parameters using a simple inverse analysis based on the global Competitive Imperialist Algorithm (CIA) [5]. In the second stage, the unknown thermal conductivity distribution was considered as an independent expansion in a series of depth-continuous functions, also with several unknown parameters. Reconstruction of these unknown parameters was done using the results obtained in the first stage and by another inverse analysis based on CIA. This 2-stage inverse algorithm was repeated while the reconstructed parameters at each stage were used as known parameters for the next stage. The iteration of the 2-stage inverse algorithm continued until the absolute value of differences between the reconstructed parameters from two iterations of the 2-stage inverse algorithm was smaller than the desired tolerance to which unknowns were calculated.

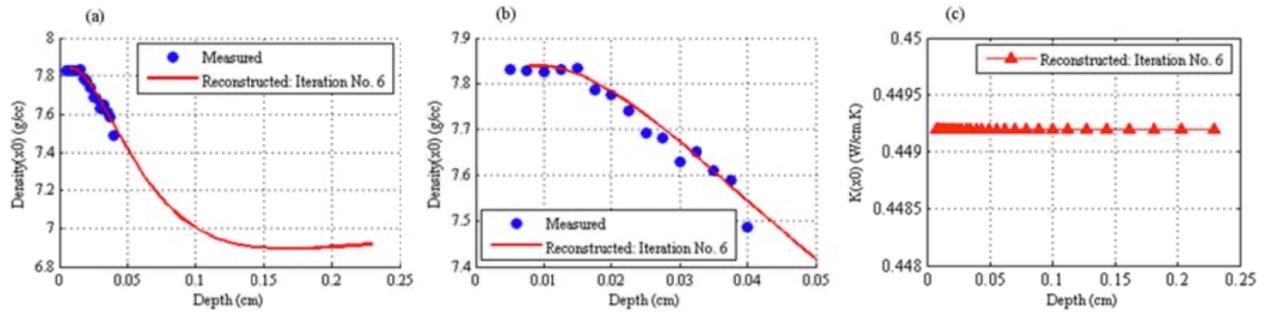
**Results and Discussion** – Reconstruction of density and thermal conductivity depth profiles was performed using a sintered metal powder compact sample with known near-surface density inhomogeneity and measuring the amplitude and phase of the surface thermal-wave field detected with the MCT detector of a conventional photothermal radiometry system [1]. The measured amplitude and



phase of the thermal-wave signal vs. frequency are plotted in Fig. 1. The amplitude response in Fig. 1 was normalized to the independently measured amplitude response of the homogeneous solid bulk using homogeneous semi-infinite thermal-wave theory [6]. Fig 2(a) indicates that the reconstructed density curve decays to a constant – bulk – value as expected from the analysis. Fig. 2(b) shows the near-surface reconstructed density depth profile along with the actual independently measured density depth profile. Fig. 2(c) shows the simultaneously reconstructed thermal conductivity depth profile, an essentially constant value equal to the bulk thermal conductivity. The inverse algorithm in Fig. 2(b) very closely reconstructed the measured density depth profile. All Fig. 2 profiles were obtained after 6 iterations of the 2-stage inverse algorithm.



**Fig. 1.** Experimental frequency data, (a) reconstructed phase difference and (b) amplitude ratio, both normalized to the bulk substrate frequency response.



**Fig. 2.** Reconstructed density profile and the actual measured density data and the reconstructed thermal conductivity profile: (a) the much deeper profile; (b) the shallow depth profile; (c): the reconstructed thermal conductivity profile

**Conclusions** – An inverse method was developed based on an integral equation representation of the inhomogeneous thermal-wave boundary-value problem which was subsequently used with a 2-stage inverse algorithm. This methodology connects the frequency scan data (amplitude and phase) with the unknown density and thermal conductivity depth profiles. Simultaneous reconstruction is an advantage of this methodology in comparison with other inverse thermal-wave problems developed to-date. This method was applied to non-destructive, non-contacting reconstruction of density and thermal conductivity depth profiles of a sintered automotive manufacturing part.

**References**

[1] M. Munidasa, F. Funak and A. Mandelis, *J. Appl. Phys.* 83:7 (1998) 3495-3498.  
 [2] S. Kooshki, A. Mandelis, A. Khodadad et al., *J. Appl. Phys.* 127:4 (2020) 045110.  
 [3] T.T.N. Lan, U. Seidel and H.G. Walther, *J. Appl. Phys.* 77(9) (1995) 4739-4745.  
 [4] T.T.N. Lan, U. Seidel, H.G. Walther, G. Goch and B Schmitz, *J. Appl. Phys.* 78:6 (1995) 4108-4111.  
 [5] E. Atashpaz-Gargari, C. Lucas, in 2007 IEEE Congress on Evolutionary Computation, IEEE (2007) 4661-4667.



[6] A. Mandelis, Diffusion-Wave Fields, Springer, New York 2001, Chap. 2.