Doppler effect for thermal waves: theory and applications

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The change in frequency of an electromagnetic wave when the source and the receiver are in relative motion one with respect to the other was described by Christian Doppler [1] in 1842. He wrongly applied the effect to explain the different colours of stars. Buys Ballot made experiments with sound waves and later Fizeau in 1848 proposed that lines in optical spectra might show variations in frequency depending upon the relative velocities of the source and the observer. The prediction was confirmed by Sir William Huggins, who showed the change in frequency towards the red of hydrogen lines recorded from Sirius, indicating a recessive motion. The effect has since then become the more and more important in the description of our world. In photoacoustics, the Doppler effect of the emitted acoustic waves from moving amplitude modulated optical sources has been explored several decades ago by Soviet researchers including Bozhkov, Bunkin, and Kolomenskii as well as by researchers in the United States, but a clear analytical approach was missing. More recently the full theory on moving photoacoustic sources in one, two, and three dimensions has been deeply discussed by W. Bai and G.J. Diebold [2] giving application to trace gas detection with dynamic photoacoustic spectroscopy (DPAS). Finally, in photothermal infrared thermography several groups have introduced the "flying spot" technique where a c.w. laser beam is moved at constant velocity for a quick inspection of the material thermal parameters and detection of cracks [3]. Here the authors showed the asymmetric thermal field profile. However, they did not investigate the Doppler effect for the induced thermal field [4]. In the present paper we give the solution for the thermal waves generated by a modulated source moving in solid media, derive the Doppler effect for thermal waves giving the expression of the frequency shift, and eventually showing some possible applications for nondestructive evaluation and testing of materials [5].

Thermal waves are wave-like temperature fields induced by a harmonic heating process. Monitoring thermal diffusion caused by a periodically modulated heat source was already used by Lord Kelvin and A.J. Ångström to investigate the thermal diffusivity of bodies, but only recently, in the early 80's, have been called "thermal waves", arousing a remarkable clamour. Presently the scientific community still debates on their very nature, even if their formalism is widely accepted to describe the temperature field in periodical regime, and most of the fundamental wave phenomena as reflection, refraction [6], interferometry [7], resonance [8] and scattering [9] have been already both theoretically and experimentally demonstrated, and used in many applications.

The differential equation of heat conduction in a moving medium has been treated for the first time in fluids by HA Wilson in 1904 as reported in the well known textbook by Carlslaw and Jaeger where the modified Fourier equation has been extended for any moving medium. The case of a plane heat source (yz plane) modulated at the angular frequency ω, and moving at constant speed $-V_S$ in the opposite direction of the x axis has been widely discussed in Ref. 5, showing that the thermal wave in the moving reference $x'=x-x_S$ can be expressed by the following analytical formula [5]

$$\begin{cases} x' \geq 0; \;\; T(x',t) = \mathrm{Re}\left\{ A \exp\left[\left(\gamma_s - \sqrt{\gamma_s^2 + 2j} \right) \dfrac{x'}{\mu} \right] \exp(j\omega t) \right\} \\ x' < 0; \;\; T(x',t) = \mathrm{Re}\left\{ A \exp\left[\left(\gamma_s + \sqrt{\gamma_s^2 + 2j} \right) \dfrac{x'}{\mu} \right] \exp(j\omega t) \right\} \end{cases} \qquad \text{Eqn.1}$$

where $A = \dfrac{I}{2e\sqrt{\omega}\sqrt{\gamma^2/2 + j}}$ and $\gamma_S = \dfrac{V_S}{\sqrt{2\omega D}}$ is the ratio between the source velocity V_S and the phase velocity of the thermal wave. But the most appealing example to observe the Doppler effect with thermal waves is looking at the distortion of both amplitude and phase wave fronts when the medium is heated by a single point harmonic heat source moving again at constant speed $-V_S$ In the 3D case the solution cannot be expressed in a simple form and should be simulated numerically (see Fig.1). It is worth nothing that in the reference at rest a Doppler shift of the angular frequency $\Delta\omega = \omega' - \omega$ can be observed as follows [5] $\dfrac{\Delta\omega}{\omega} = \pm\gamma_{o,s} \cdot \sqrt{\dfrac{1}{2}\left(\sqrt{\gamma_s^4 + 4} - \gamma_s^2\right)}$ where $\gamma_{o,s} = V_{O,S}/\sqrt{2\omega D}$ and $V_{O,S}$ is the relative speed when both observer and source are moving away from each other, and the plus sign is for the reciprocal approach.

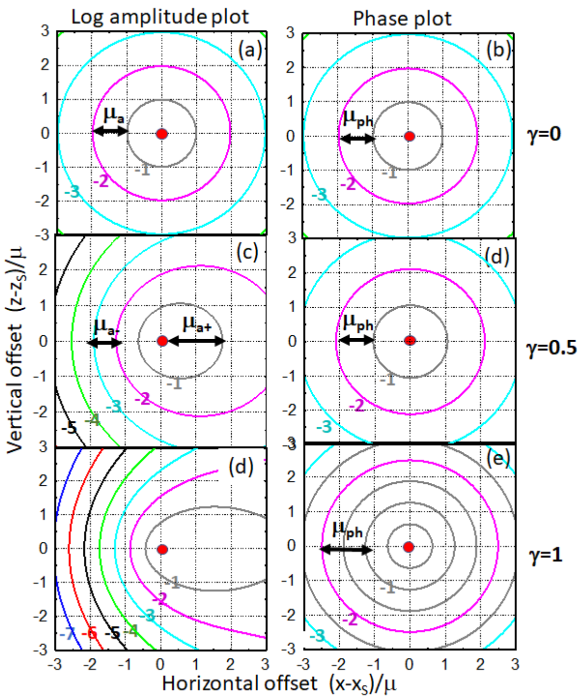

Fig. 1. Contour plot of the induced temperature field in the reference frame of the source. Modulated heat source is in the origin. Both coordinates are dimensionless because normalized to the thermal diffusion length. (a),(c),(d) Logarithmic amplitude contour plot respectively for γ_S=0, 0.5, and 1; (b),(d),(e) phase contour plot respectively for γ_S =0, 0.5, and 1.

References

[1] C Doppler, Goesellschaft d. Wiss. 2, 465, (1842).

[2] W. Bai and G. J. Diebold, Phys. Rev. E, 98, 032125 (2018).

[3] Bedoya, J. González, J. Rodríguez-Aseguinolaza, A. Mendioroz, A. Sommier, J.C. Batsale, C. Pradere, A. Salazar, Measurement 134, 519–526 (2019).

[4] M. Colom et al., Int. J. Therm. Sci. 151, 106277 (2020).

[5] R. Li Voti, M. Bertolotti, International Journal of Heat and Mass Transfer, 176, 121098 (2021)

[6] Bertolotti. M, G.L.Liakhou, R. Li Voti, S.Paoloni, C. Sibilia, J.Appl.Phys 85, 3540 (1999).

[7] R. Li Voti, et al, Journal of Optoelectronics and Advanced Materials 3, 779 (2001).

[8] M. Bertolotti, G.L. Liakhou, R.L Voti, S.Paoloni, C. Sibilia, Int. J. Thermophys. 19, 603 (1998).

[9] R. Li Voti, C.Sibilia, M.Bertolotti, Int. J. Thermophys. 26, 1833 (2005).