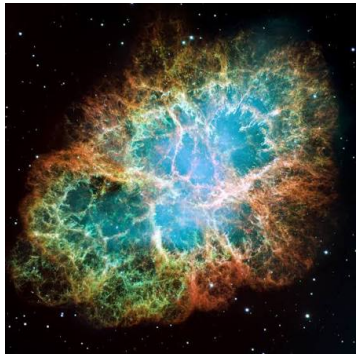


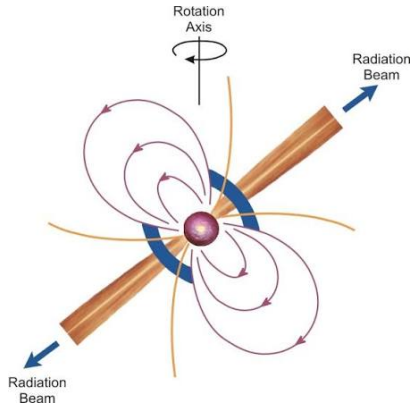
Luminosity function of millisecond pulsars in globular clusters from their gamma-ray spectra

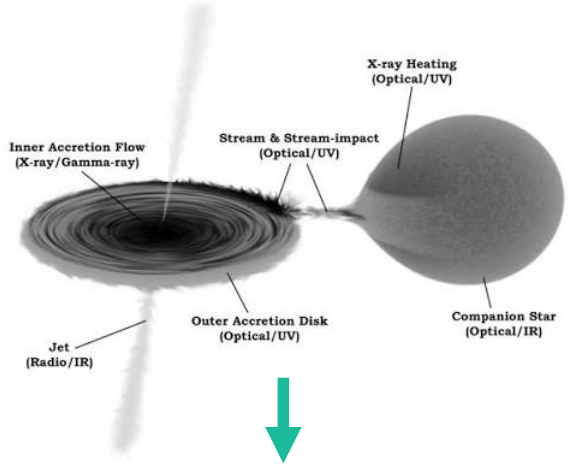
Francesca Calore (LAPTh), Christopher Eckner (IAC, La Laguna), Silvia Manconi (LPTHE, Paris), Francesco Xotta[†] (UNG), Gabrijela Zaharijas (UNG)

[†]Speaker: francesco.xotta@ung.si

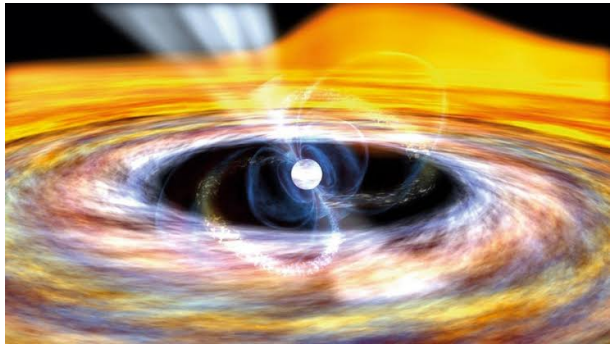
How do stars die and come back to life ?







- During its evolution, a massive star can “die” and transform into a Neutron Star.
- A Neutron Star can encounter another star or be born in a binary system. There, it accretes material and speeds up massively, reigniting and “coming back to life”. Eventually, we have a Millisecond Pulsar (MSP).



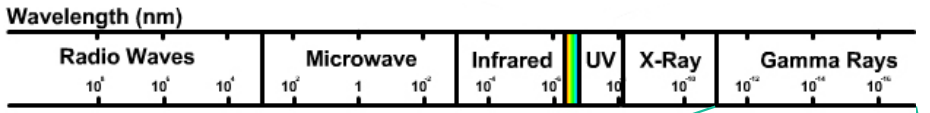
Figures from: https://science.nasa.gov/asset/hubble/a-giant-hubble-mosaic-of-the-crab-nebula/, https://www.nrao.edu/pr/2005/terzan5/, https://www.researchgate.net/figure/Geometry-of-a-Roche-lobe-overflow-low-mass-X-ray-binary-The-color-scale-is-inverted-with_fig1_47546646, https://physicsworld.com/a/magnetic-fields-put-the-brakes-on-millisecond-pulsars/

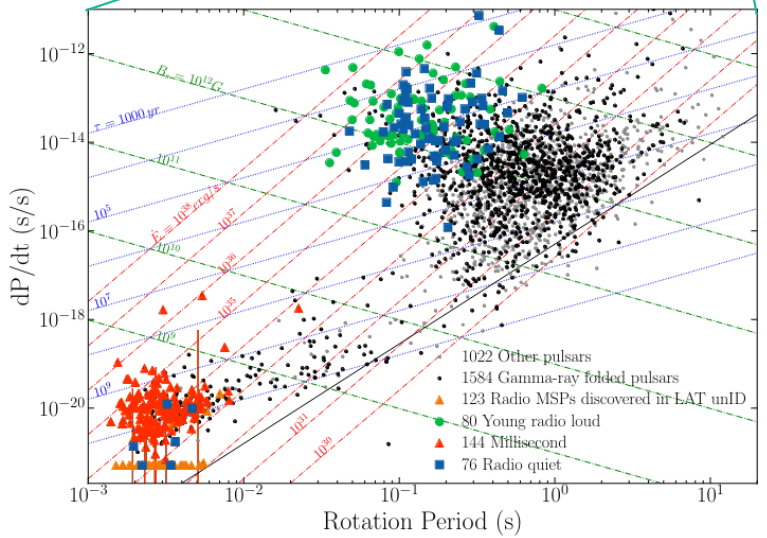
What are MSPs?

MSPs are old highly magnetized Neutron Stars with rotational Period <30 ms

- Observed for the first time in radio [1]
- We know around 300 MSPs, ~144 of which detected in gamma-rays

Determining their luminosity distribution is crucial





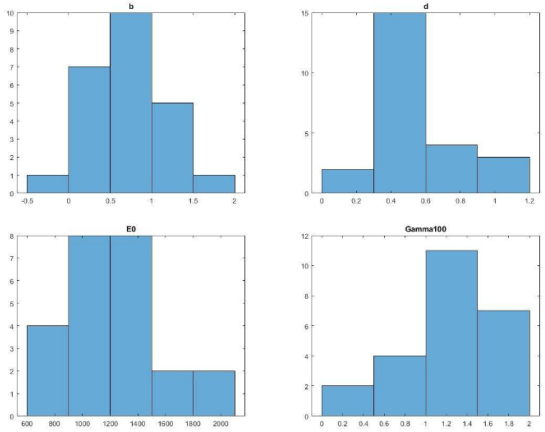
Smith et al., 2307.11132

[1]: Bhattacharyya & Roy, arxiv: 2104.02294

MSPs Gamma-Ray Spectrum

- Pulsars' gamma-ray Spectral Energy Distributions (SEDs) are well represented by the Power Law with Exponential Cutoff (PLEC) distribution, given as:

$$\frac{dN}{dE} = N_0 \left(\frac{E}{E_0}\right)^{-\Gamma+\frac{d}{b}} \exp\left(d/b^2\left(1-\left(\frac{E}{E_0}\right)^b\right)\right)$$



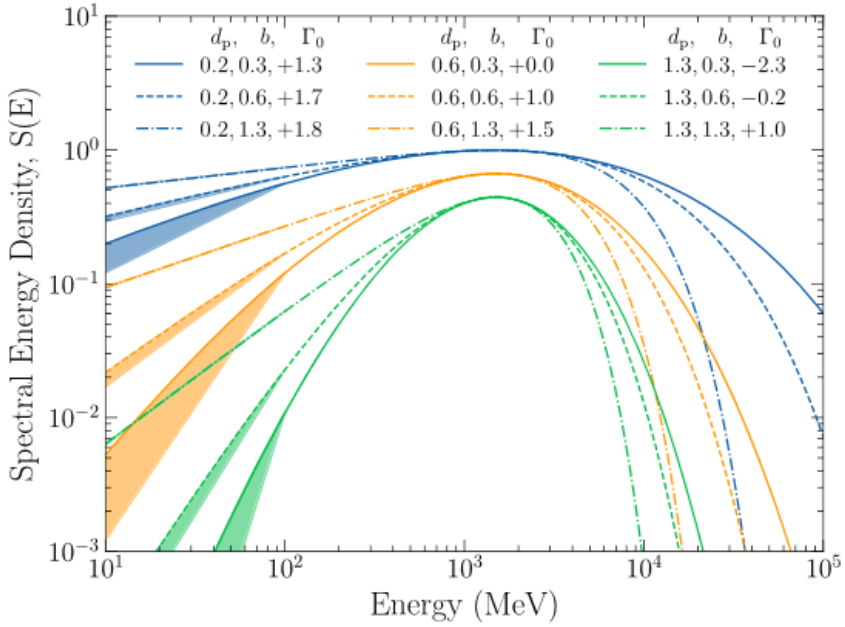
Mean Values for MSPs:

d = 0.6 ± 0.3

b = 0.7 ± 0.5

Γ = 1.9 ± 0.8

E_0 = (1.2 ± 0.3) GeV



Smith et al., arxiv: 2307.11132

MSPs and Globular Clusters

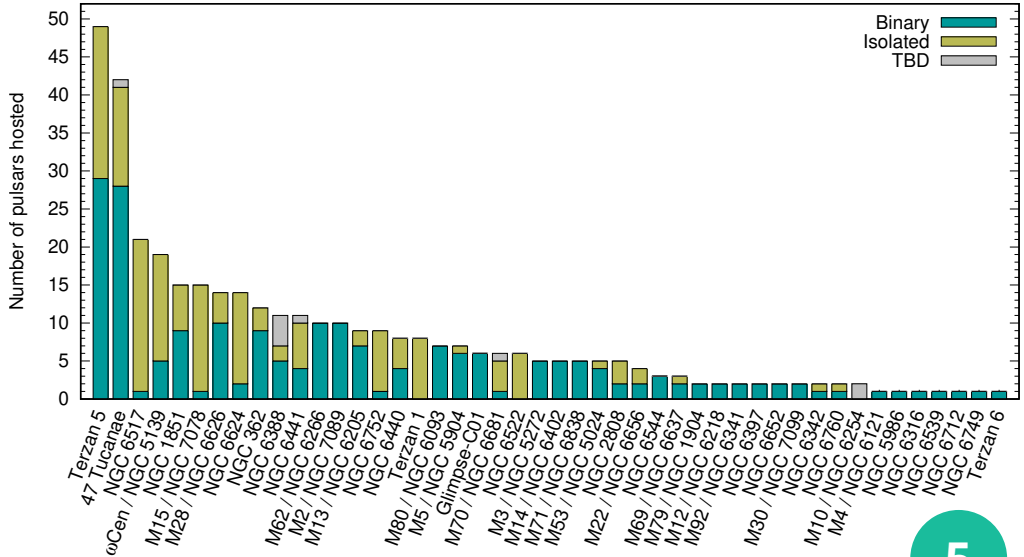
Globular Clusters (GCs) are old spheroidal conglomeration of stars bound by gravity

- MSPs make up most of the gamma-ray emission from GCs
- Most (~40%) MSPs are found in GCs through radio and X
- The formation of binaries in GCs is likely through tidal capture

What can we say about MSPs inside of GCs?



369 pulsars in 46 clusters



Terzan5 photo from Hubble

Plot from https://www3.mpifr-bonn.mpg.de/staff/pfreire/GCpsr.html

This Project

Goal:

- Derive the luminosity function of MSPs in GCs using only gamma-ray emission from GCs, as observed by the *Fermi* Large Area Telescope (*Fermi*-LAT)

Method:

Compare simulated to real spectra using:

- Nested Sampling
- Simulation Based Inference

Members:

- Francesca Calore (LAPTh)
- Christopher Eckner (UNG, IAC)
- Silvia Manconi (LAPTh, LPTHE)
- Francesco Xotta (UNG)
- Gabrijela Zaharijaš (UNG)

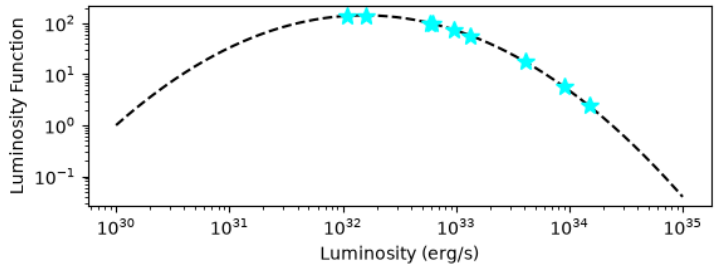


Simulating GCs Fluxes

Given a parameter set (L_0, σ_L, N_l), l=1,...,N_{GC}:

- For each GC, sample N_l MSP luminosities L_c:

$$\frac{dN}{dL_\gamma} = \frac{1}{L_\gamma \sigma_L \sqrt{2\pi}} \exp\left(-\frac{(\ln L_\gamma - \ln L_0)^2}{2\sigma_L^2}\right)$$

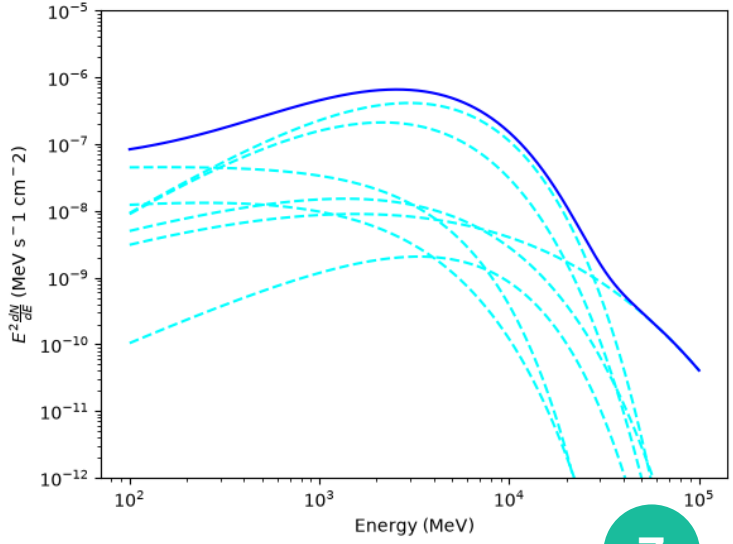


- Sample MSPs spectral parameters from PLEC distribution, find normalizations:

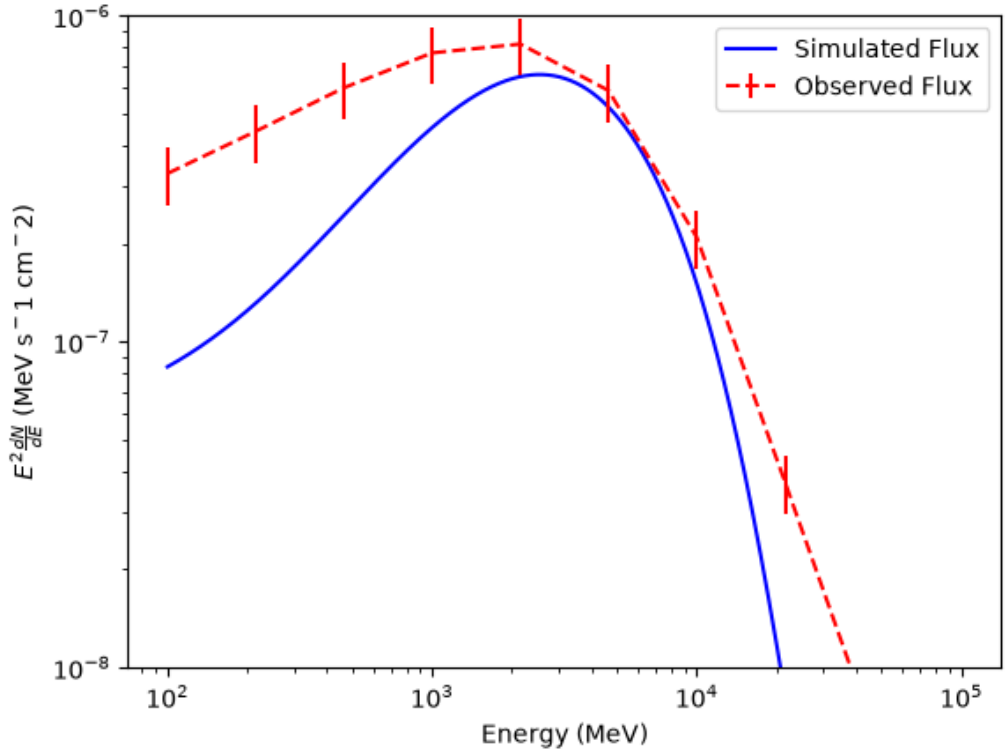
$$F_{\mathrm{MSP,c}} \equiv \int_{0.1\ \mathrm{GeV}}^{100\ \mathrm{GeV}} k_c \times \frac{dN}{dE}(E) \times E\, dE \equiv \frac{L_I}{4\pi {d_{GC}}^2}$$

- Cumulative spectrum from all MSPs:

$$F^k_{\mathrm{MSP}} = \sum_{c=1}^{N} \int_{E_k-\Delta E_k/2}^{E_k+\Delta E_k/2} k_c \times \left.\frac{dN}{dE}\right|_c (E) \times E\, dE$$



Likelihood Based Analysis



Sample N_I luminosities $I=1,\ldots,N_{GC}$

Find simulated spectrum

Define χ^2:

$$\chi^2 = \sum_k \left(\frac{F_{\mathrm{MSP},I} - F_{\mathrm{GC},I}}{\Delta F_{\mathrm{GC},I}} \right)_k^2$$

Contribution for GCs with 1 known MSP:

$$\chi^2_{max} = \sum_k \left(\frac{(F_{\mathrm{MSP},I,max}) - F_{\mathrm{GC,I}}}{\Delta F_{\mathrm{GC,I}}} \right)_k^2$$

For each parameter set θ

Minimize χ^2

Nested Sampling

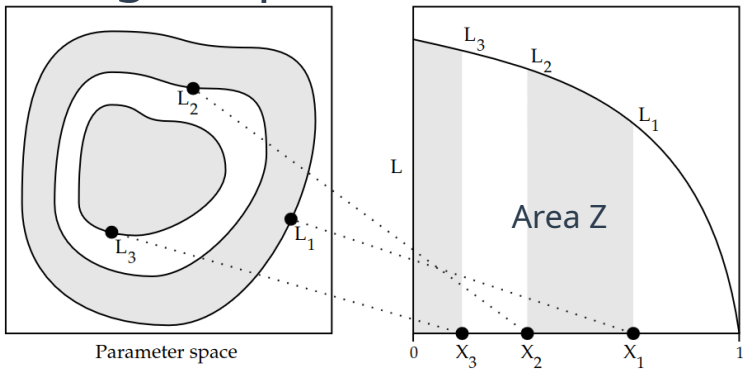
Given the parameter set $x=(L_0, \sigma_L, N_I)$ and the associated fluxes $z=(F_{MSP,I})$

$$p(x|z)=\frac{p(z|x)\,p(x)}{p(z)}$$

The objective is to estimate Z:

$$Z=\int p(z|x)\,p(x)\,dx=\int p(z|x)\,dX$$

Which we do by a trapezoidal using the prior mass X



Find point c with least likelihood L_c

⇩

Estimate X_c roughly as $X_c = \exp(-c / N)$
Find weight $w_c = (X_{c-1} - X_c)/2$

⇩

Estimate $Z=Z + L_c \cdot w_c$

Repeat until Z reaches the desired value

Validation with Synthetic Data: A Matter of Speed

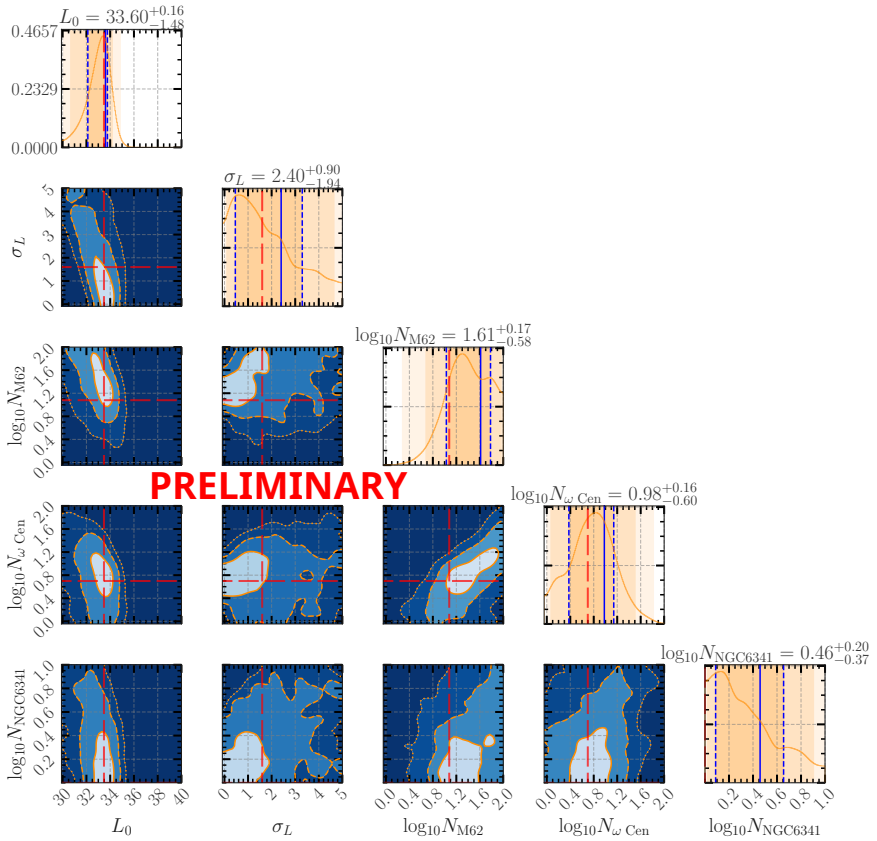
We test our method with synthetic data, observing that:

- L_0, σ_L are recovered at 1σ
- The numbers of MSPs are recovered at 2σ

Time is a limiting factor:

- ~0.4 hours for 1 GC
- ~4.6 hours for 2 GCs
- >15.9 hours for 3 GCs

We need something faster!

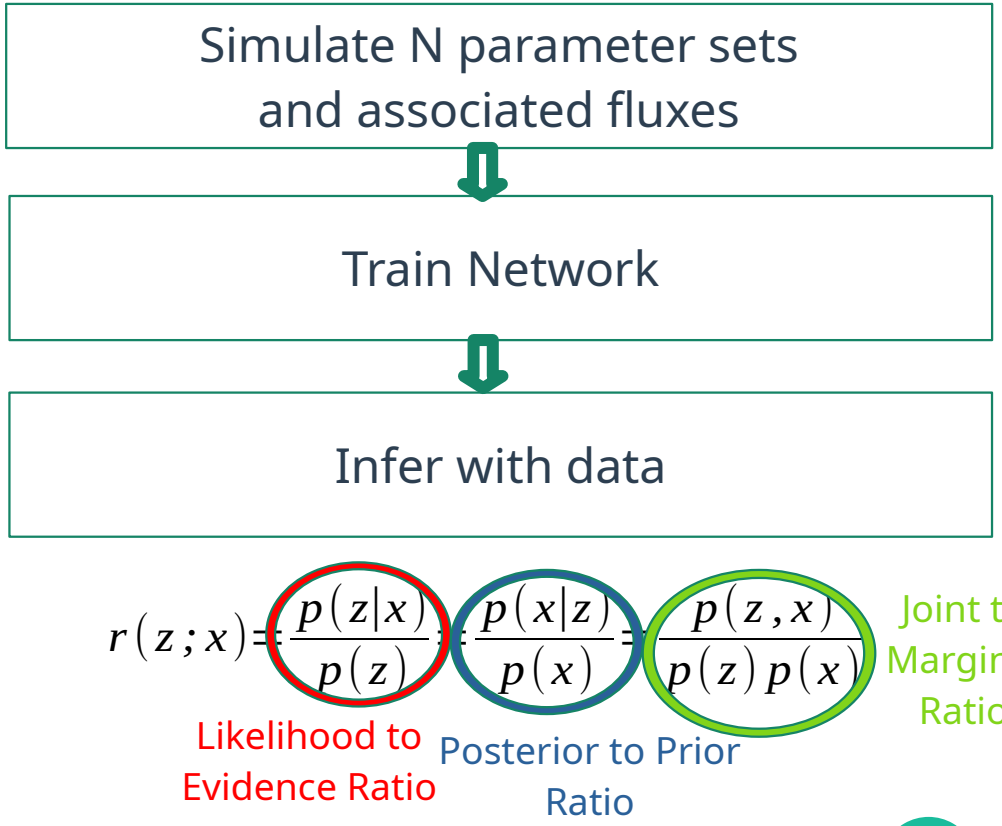


Implicit Likelihood Analysis: Neural Ratio Estimation

Implicit Likelihood Method:

- Run the simulator for N parameter sets, obtaining a flux in the same way as the likelihood based method
- Given the parameter set $x=(L_0, \sigma_L, N_I)$ and the fluxes $z=(F_{MSP,I})$, $I=1,\ldots,N_{GC}$, train a Neural Network to classify whether or not $(x, z)\sim p(z,x)=p(z|x)p(x)$ → *Given (x,z), did x generate z?*
- Marginalize posteriors using data

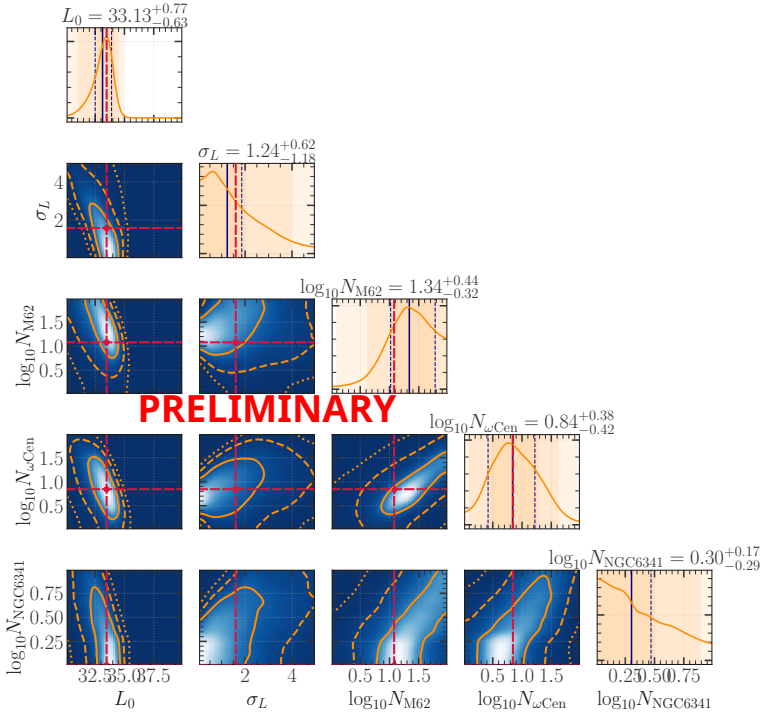
We implement Simulation Based Inference through Swyft [3]



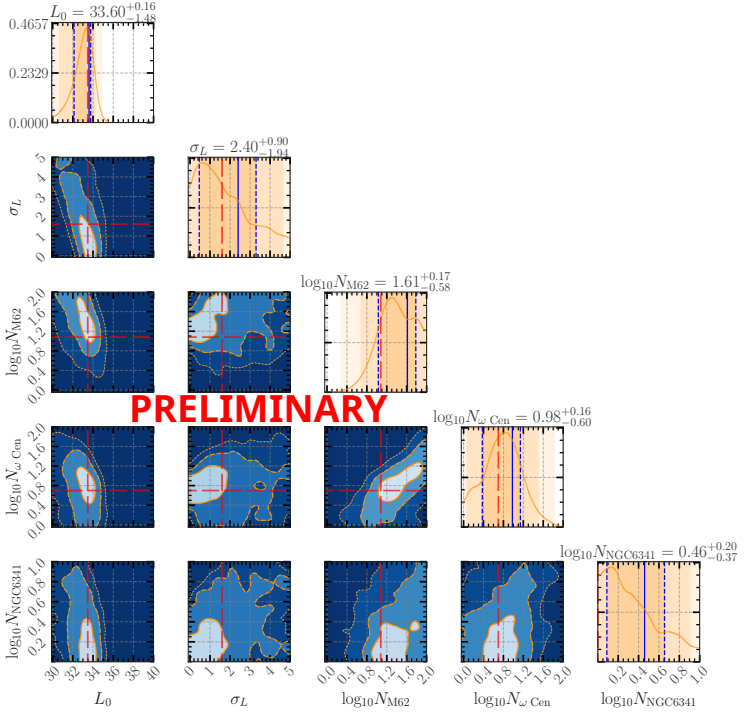
[3]: Kurt et al., arxiv: 2011.13951

Likelihood implicit comparison to traditional methods

SBI



Nested Sampling

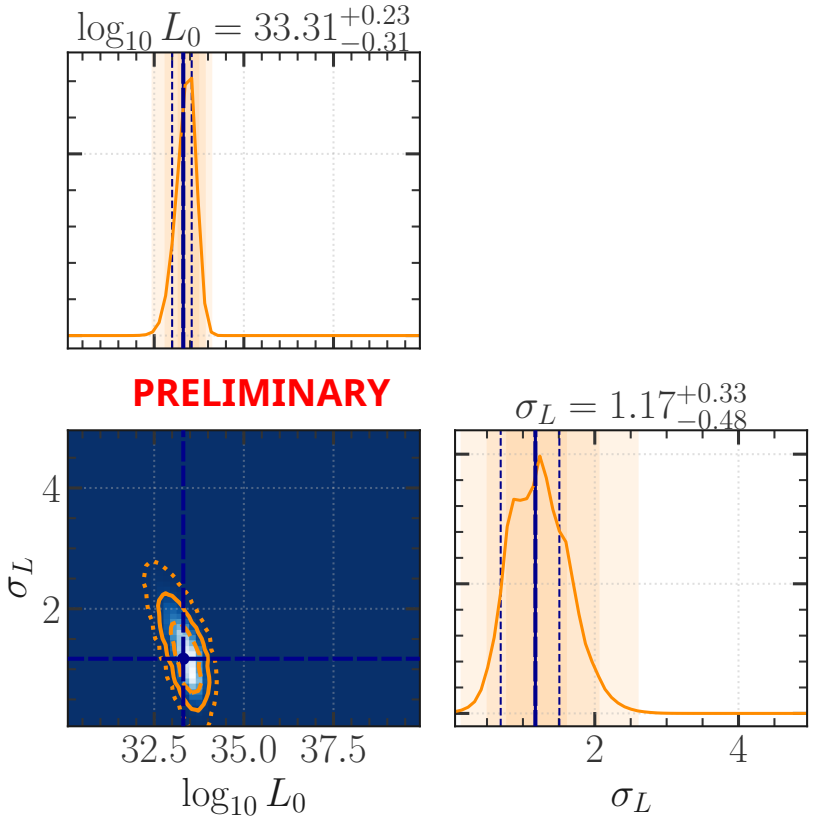


SBI and Nested Sampling show similar marginal posteriors

SBI Results with Real Data

We run SBI over all real GC samples and marginalize over the luminosity function parameters.

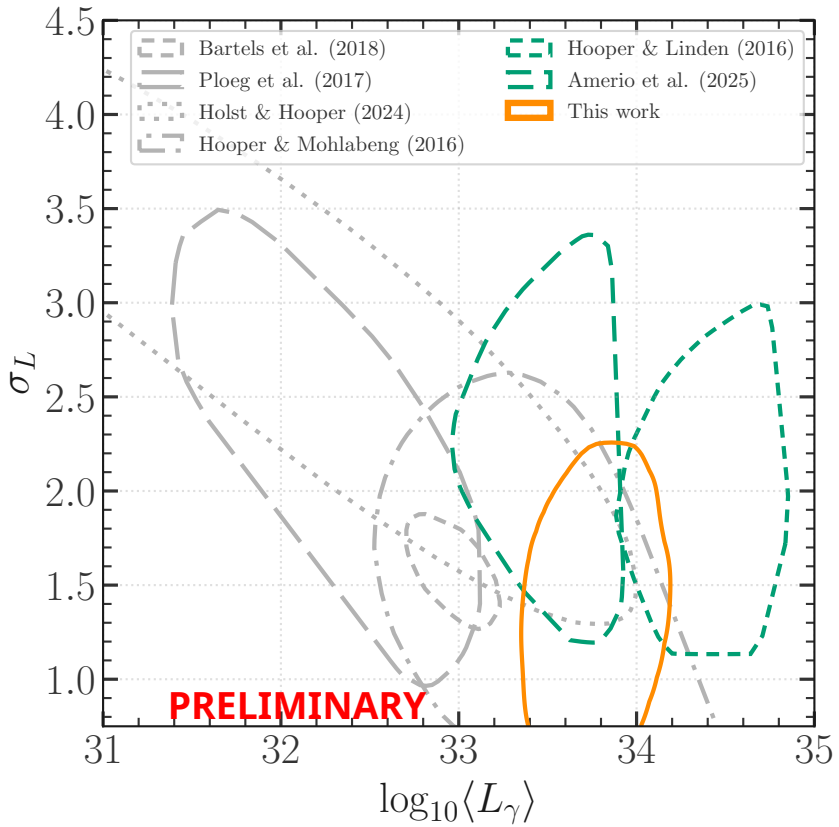
- SBI does not need to estimate the posterior for all parameters involved
- SBI can produce well behaved posteriors even considering our full GCs data set



Comparison to past studies

We compare 95% confidence levels of our results and available literature for MSPs in GCs and Field MSPs. Our results:

- Don’t assume any specific model (no relation with stellar encounter rate, no model on MSPs formation)
- Only use spectral information (no morphological information)
- Only use gamma-ray information (no X-Rays or Radio information)



Conclusion

- **We derived the luminosity function of MSPs in GCs relying only on gamma-ray spectral data.**

  Our results are consistent with the literature for MSPs in GCs, but don’t rely on complementary data, which could introduce biases.

- Traditional Likelihood Based methods are shown to be unfeasible for our real data sets due to the extensive runtime required.

  However, **the adoption of SBI, which is shown to be much faster than Nested Sampling, makes the analysis of our real data sets possible**. We validate its results by comparing with the traditional likelihood based technique we adopted.

- **Our study: soon to be submitted :)**

Thanks for Following!

GCs Data Used in this Study

Datasets:

- 7 brightest GCs (‘Bright’)
- 25 GCs, adding intermediate brightness GCs to the Bright data set (‘Intermediate’)
- All 36 4FGL Sources firmly associated to known GCs (‘Baseline’)
- 44 4FGL Sources within 0.5° from known GCs (‘Extended’)
- 8 Mock GCs (‘Synthetic’)

Special Additional Clusters:

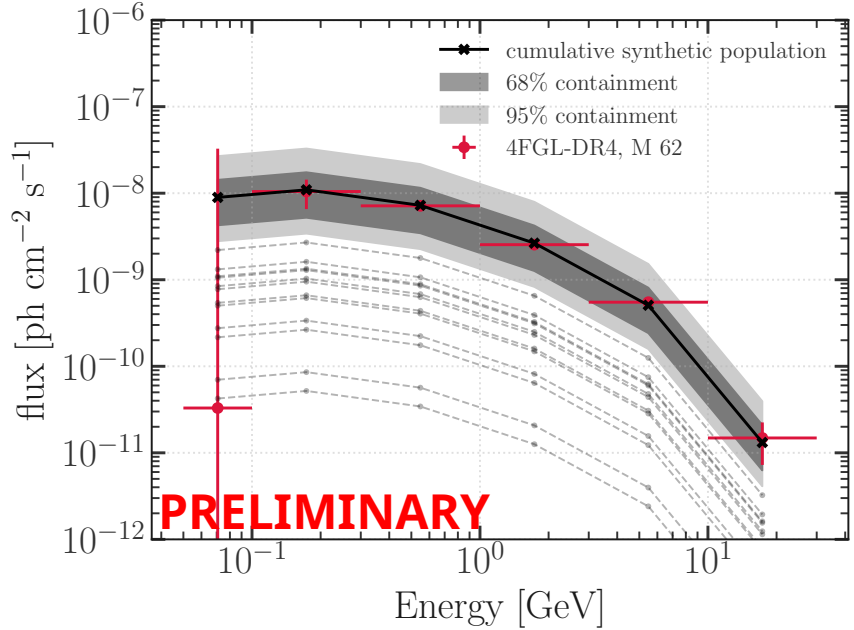
- GCs with 1 MSP + significant off-phase emission: NGC 6624, NGC 6626 (Our Analysis of LAT data). These are added to all previous datasets.

Synthetic GCs Data

Simulated GCs spectra:

Starting from 4FGL-DR4 [4] SEDs and for fixed L_0, σ:

- Simulate for each GC 500 synthetic populations of MSPs
- Fix the number of MSPs for each GC to a value that generates an SED that encompasses at 68% the 4FGL-DR4 uncertainty



[4]: Ballet et al., arxiv: 2307.12546

Derivation of spectra of NGC 6624 and NGC 6626: data preparation

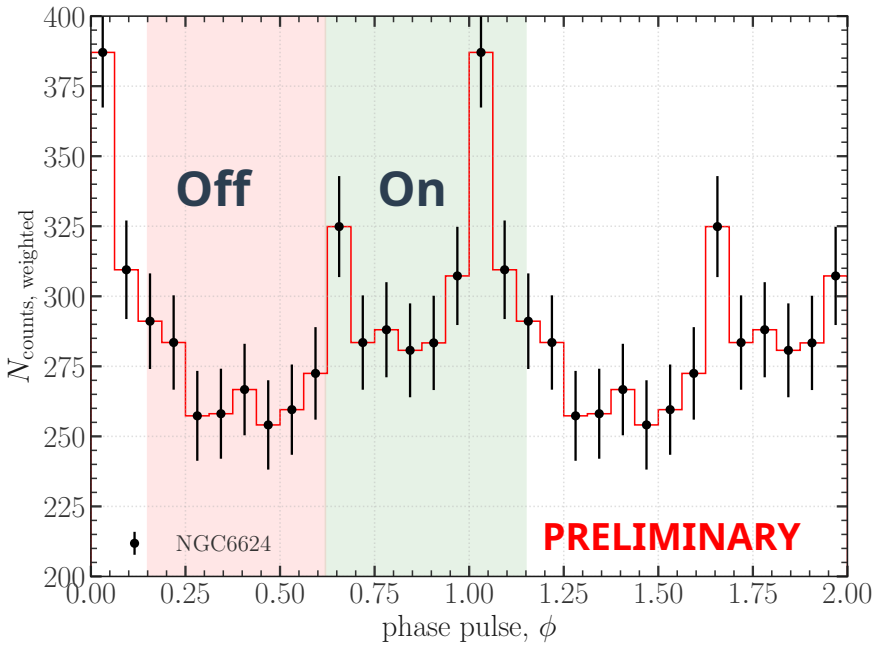
Data Selection and Preparation:

- 17 years of *Fermi*-LAT events (2008 August 4, 15:43:36.0 (UTC) and 2025 December 14, 15:42:56.0 (UTC))
- 0.05 GeV – 100 GeV
- Search Radius: 14° around the GC
- RoI: 17°x17° around the GC

Models for RoI emissions:

- Isotropic Diffuse (iso_P8R3_SOURCE_V3_v1)
- Galactic Diffuse (gll_iem_v07)
- 4FGL-DR4 Catalog (gll_psc_v35)

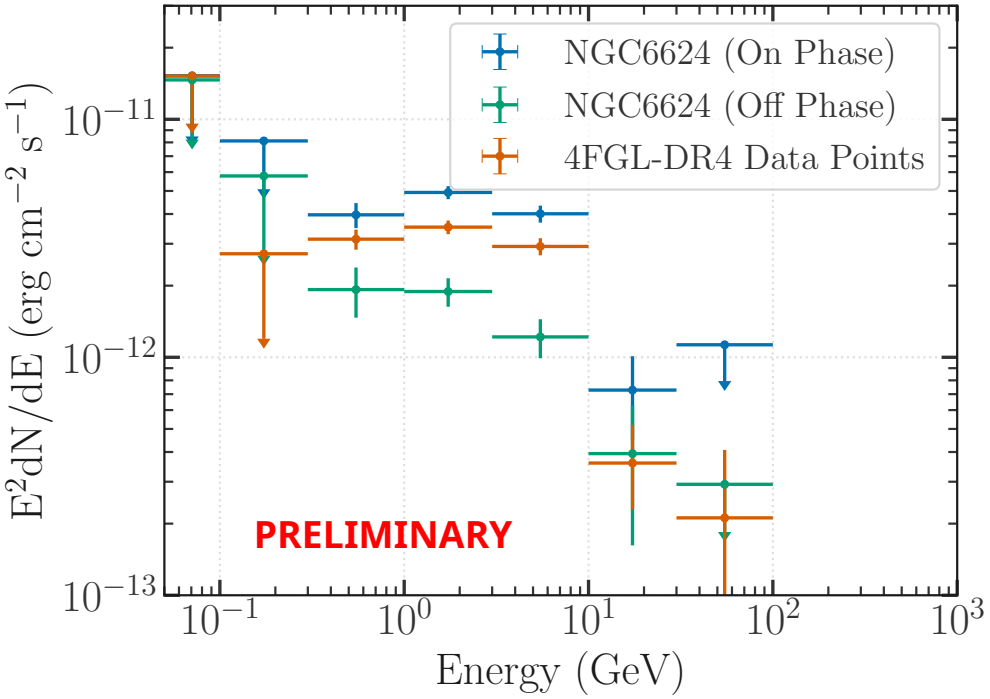
Define ON-Phase and OFF-Phase Regions using Tempo2 with *Fermi* Plug-In

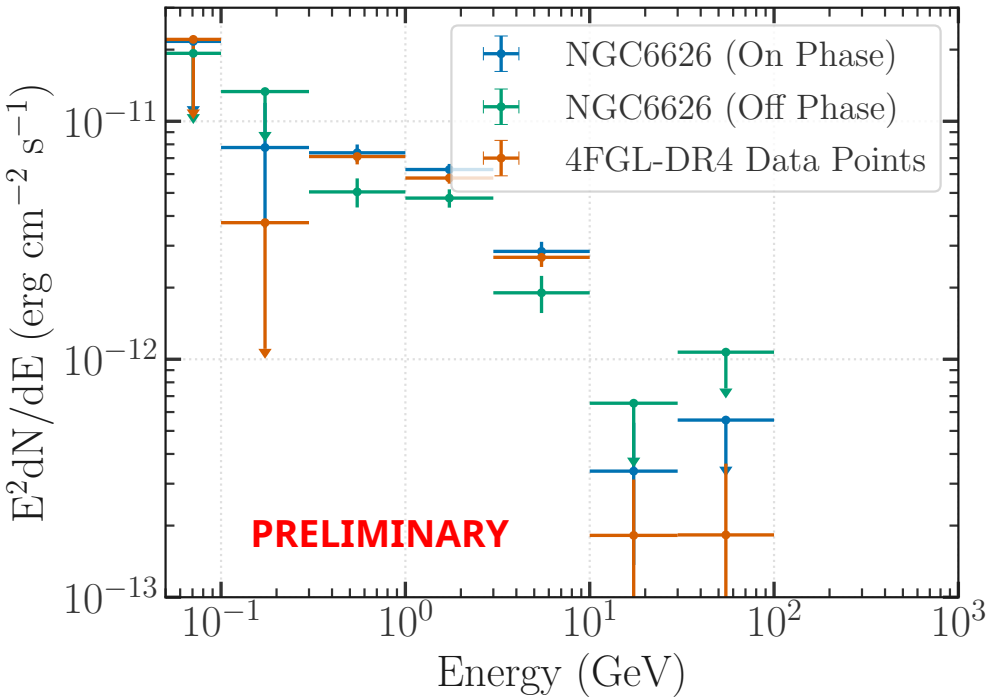


Off-Phase emission first found in Zhang et al., arxiv: 2301.12697v2

Ephemerides from https://fermi.gsfc.nasa.gov/ssc/data/access/lat/ephems

Derivation of spectra of NGC 6624 and NGC 6626: SEDs





The ON phase SED is expected from 1 single MSP, while the Off phase is expected to come from any number of remaining MSPs in the GC