Quantum Information Meets Statistical Inference

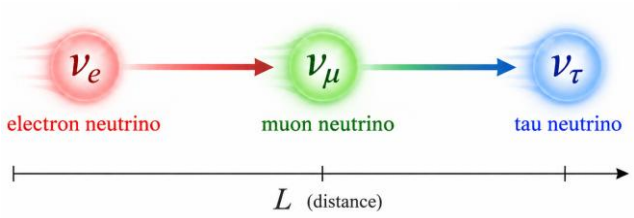
New Perspectives on Precision Neutrino Physics

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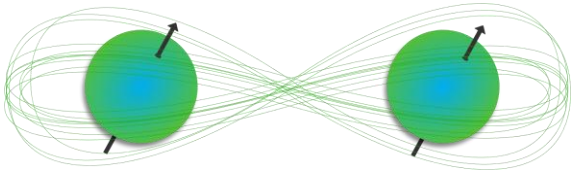


Motivation

Neutrino Oscillations:



Quantum correlations:



Earlier

- Neutrino as potential candidates for transmitting QI, low-level communication demonstrated by *Stancil et al. 2012*.
- Stands good at the test of *entanglement* and *nonlocality, Formaggio et al. 2016, Fu et al., 2017*.

Now

- Open problem in neutrino oscillation sector
- Motivates using QI diagnostics as an additional lens complementary to standard oscillation analyses, to identify where the quantum state is most informative.



Neutrino Oscillations

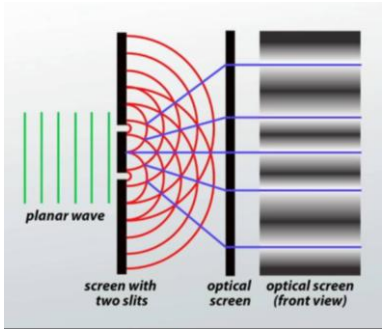
- Flavor states $|\nu_\alpha\rangle$ are superposition of mass eigenstates $|\nu_i\rangle$ and vice-versa as

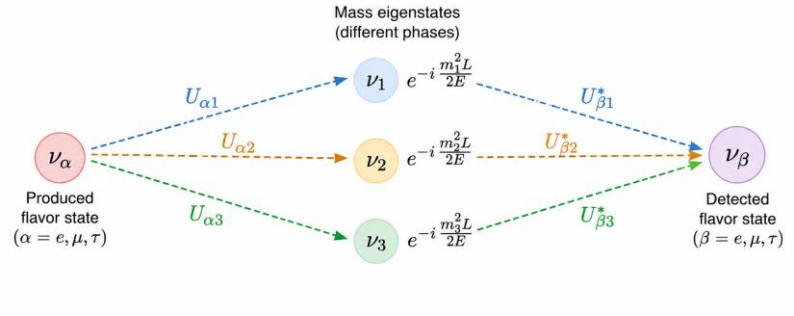
$$|\nu_\alpha\rangle = U_{PMNS}(\theta_{12},\theta_{23},\theta_{13},\delta)|\nu_i\rangle$$

- Time evolution of the flavor states

$$i\frac{\partial}{\partial t}|\nu_\alpha(t)\rangle = H_f|\nu_\alpha(t)\,;$$
$$|\nu_\alpha(t)\rangle = e^{-iH_f t}|\nu_\alpha(0)\rangle\,;$$
$$H_f = UH_mU^{-1};\ H_m = diag(E_i)$$

- The Hamiltonian may describe propagation of neutrinos in vacuum and/or in a potential (matter effect).
- Flavor oscillation probabilities





$$P_{\nu_\alpha\to\nu_\beta} = \delta_{\alpha\beta} - 4\sum_{j>k}\mathrm{Re}\left(U^*_{\alpha j}U_{\beta j}U_{\alpha k}U^*_{\beta k}\right)\sin^2\left(\frac{\Delta m^2_{jk}L}{4E}\right) + 2\sum_{j>k}\mathrm{Im}\left(U^*_{\alpha j}U_{\beta j}U_{\alpha k}U^*_{\beta k}\right)\sin\left(\frac{\Delta m^2_{jk}L}{2E}\right)$$

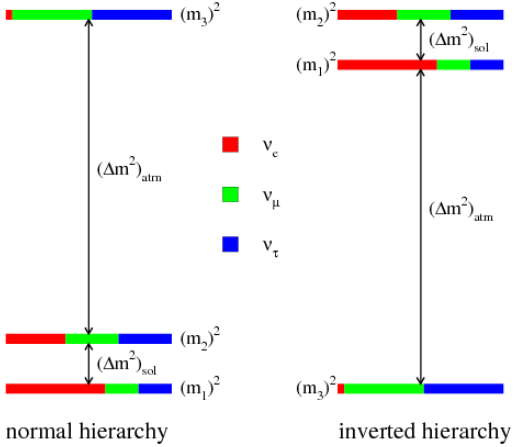
$$\Delta m^2_{jk} \equiv m^2_j - m^2_k$$

Oscillation Parameters: Status

- Many parameters have been measured with good accuracy: θ_{12}, θ_{13}, Δm^2_{21} and Δm^2_{31}.
- Parameter θ_{23} is measured with relatively large uncertainties.

Open problems in 3-flavor oscillation:

- CP violation: $\delta \neq 0 \implies P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$
- θ_{23}-octant: lower (<45^o) or higher (>45^o)?
- Neutrino mass hierarchy: whether $m_1 \leq m_2 \leq m_3$ or $m_3 \leq m_1 \leq m_2$
- ***Precise measurement***



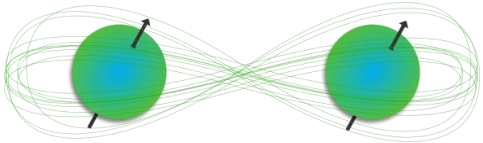
These unresolved questions motivate asking where, in energy and baseline, the neutrino state carries the most useful information about the parameters we want to measure.

Entangled Flavor States of Neutrino

- *Separable states, not entangled:* $|1\rangle_A \otimes |0\rangle_B \equiv |01\rangle_{AB}$

$$\frac{1}{2}\left(|00\rangle_{AB} + |01\rangle_{AB} + |10\rangle_{AB} + |11\rangle_{AB}\right) = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)_A \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)_B$$

- ***Entangled states:*** $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$

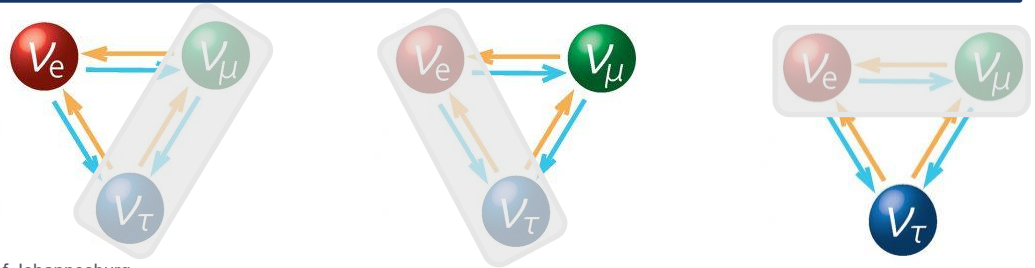


- ***Neutrino flavor state:*** $|\nu_\alpha(t)\rangle = a_{\alpha e}(t)|\nu_e\rangle + a_{\alpha\mu}(t)|\nu_\mu\rangle + a_{\alpha\tau}(t)|\nu_\tau\rangle$

Occupation number representation: $|\nu_e\rangle \equiv |1\rangle_e|0\rangle_\mu|0\rangle_\tau$; $|\nu_\mu\rangle \equiv |0\rangle_e|1\rangle_\mu|0\rangle_\tau$; $|\nu_\tau\rangle \equiv |0\rangle_e|0\rangle_\mu|1\rangle_\tau$

$$|\nu_\alpha(t)\rangle = a_{\alpha e}|1\rangle_e|0\rangle_\mu|0\rangle_\tau + a_{\alpha\mu}|0\rangle_e|1\rangle_\mu|0\rangle_\tau + a_{\alpha\tau}|0\rangle_e|0\rangle_\mu|1\rangle_\tau$$

Flavor-mode entanglement



$|1\rangle_\alpha \rightarrow$ *Presence in* α-mode

$|0\rangle_\alpha \rightarrow$ *absence in* α-mode

Precision Neutrino Physics: The Problem

- **NOνA** (L = 810km, E ~ 2GeV) and **T2K** (L = 295km , E ~ 0.6GeV), accelerator neutrino experiments, probe the same 3-flavor framework but operate in different L and E regimes.
- Their results show tension in δ_{CP} and θ_{23}-octant preference.
- Traditional fits ask which oscillation parameters reproduce event spectra.
- QI lens: where is the flavor state closest to a parameter-sensitive, nearly separable configuration?

Total concurrence

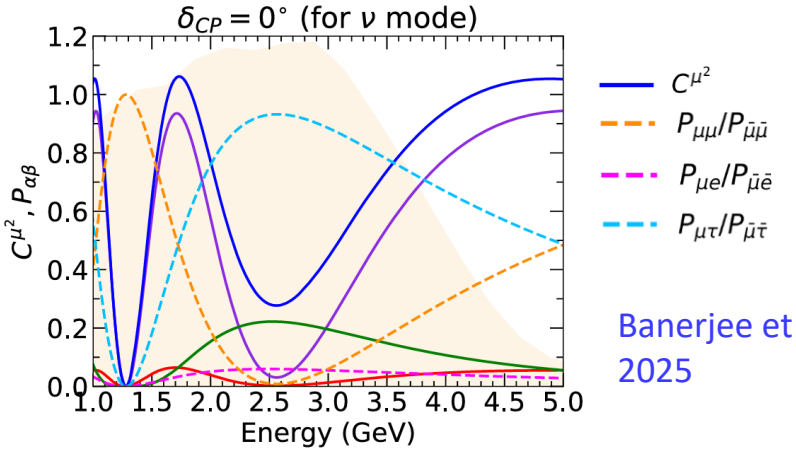
Measures entanglement among flavor modes in a single-neutrino state.

Coffman et al., 2000

$$C^{\mu^2} = C^2_{e|\mu\tau} + C^2_{\mu|e\tau} + C^2_{\tau|e\mu}$$

$$C^2_{e|\mu\tau} = 2(1 - Tr\rho_e^2) = 4P_{\mu e}(1 - P_{\mu e})$$

Local minima identify energy windows where the flavor state is closest to separability.



Banerjee et al., 2025

Where the QI Diagnostics Point

To reduce the effect of different operational energy ranges, we choose common oscillation parameter sets that minimize total concurrence for T2K and NOvA within their respective allowed FWHM ranges around peak fluxes.

Type of extrema	Set	Experiment [E (GeV)]	$\Delta m^2_{21}/(10^{-5})$ [eV^2]	θ_{12}	θ_{13}	θ_{23}	$\Delta m^2_{31}/(10^{-3})$ [eV^2]	δ_{CP}	Total concurrence (NMO)
Min	Set-1	NOνA [1.73]	8.05	35.9	8.19	45.0	2.574	88	0.165057
		T2K [0.62]							0.131989
	Set-2	NOνA [2.01]	6.93	35.9	8.19	45.0	2.574	92	0.40973
		T2K [0.685, 0.75]							0.260415 (0.447701)

For NMO

For IMO

Type of extrema	Experiment [E (GeV)]	$\Delta m^2_{21}/(10^{-5})$ [eV^2]	θ_{12}	θ_{13}	θ_{23}	$\Delta m^2_{31}/(10^{-3})$ [eV^2]	δ_{CP}	Total concurrence (IMO)
Min	NOνA [1.93–2.03]	8.05	35.9	8.25	45.0	-2.547	88	0.216–0.319
	T2K [0.62]							0.0998

KD, Kundu, Panda, Razzaque, Sarkar, 2026

Precision Neutrino Physics: The Problem

Given a physical parameter α, conditional probability p(x|λ) of observing the classical measurement outcome x and a set of measurement outcomes x = {x_1,...,x_M}, the achievable precision of any unbiased estimator **α**

Fisher information

$$F_C(\alpha) = \sum_x \frac{1}{p(x|\alpha)} \left[\frac{\partial p(x|\alpha)}{\partial \alpha}\right]^2$$

Classical Cramér-Rao bound

$\mathrm{Var}(\alpha) \geq 1 / (N \cdot F_C)$

Quantum Fisher information: Quantifies intrinsic information about a parameter in the quantum state.

$$H_Q(\alpha) = 4\left(\langle \partial_\alpha \psi_\alpha | \partial_\alpha \psi_\alpha \rangle - |\langle \psi_\alpha | \partial_\alpha \psi_\alpha \rangle|^2\right).$$

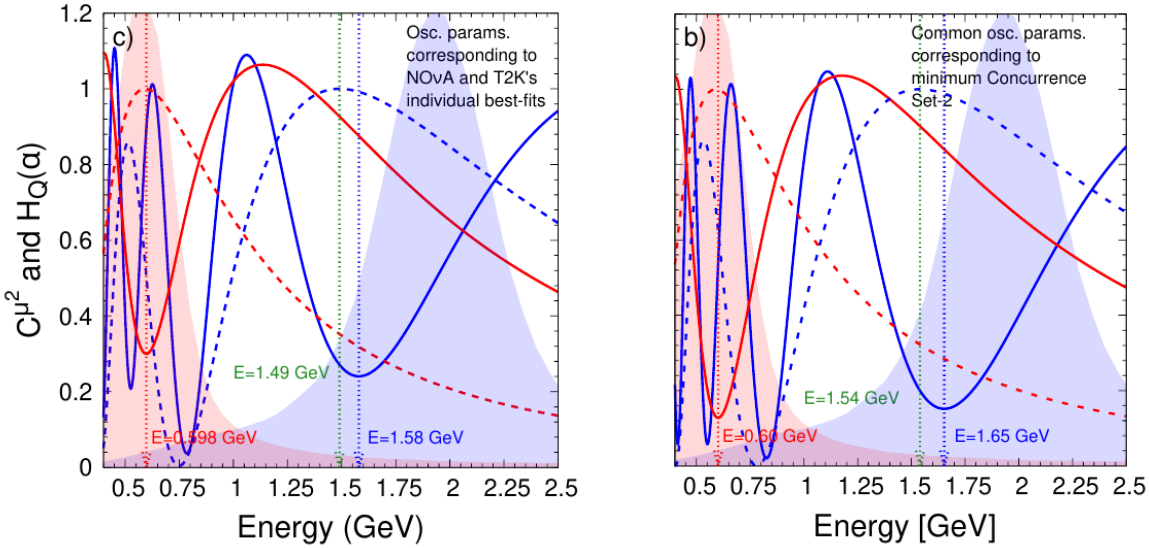
$\mathrm{Var}(\alpha) \geq 1 / (N \cdot F_C) \geq 1 / (N \cdot H_Q)$

The quantum Cramér-Rao bound, ultimate precision limit on what any measurement could achieve.

Helstrom, 1969

- **F_C:** How much a GIVEN measurement tells you about α, depends on the observable you chose.
- **H_Q:** How much information is available, maximum over all possible measurements; a property of the state alone.

Where the QI Diagnostics Point



Key reading

-Solid curves: total concurrence

-Dashed curves: QFI for $\alpha = \delta_{CP}$

-Shaded regions: experimental flux from T2K and NOνA

-Vertical lines: local minima / QFI maxima

The interesting region is where intrinsic quantum sensitivity and experimental statistics overlap.

LMS Strategy: choosing such common oscillation parameter sets induces concurrence local-minima-shift **(LMS)** and nearby QFI maxima shift toward the higher-event-count energy region.

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Statistical Analysis: Likelihood-Based Sensitivity

Poisson likelihood-based

$$\chi^2 = \min_{\vec{\lambda},\vec{\kappa}} \left[2 \sum_{i=1}^{n} \left(\tilde{y}_i - x_i - x_i \ln \frac{\tilde{y}_i}{x_i} \right) + \kappa_s^2 + \sum_l \kappa_{b,l}^2 \right]$$

Event prediction

x_i: observed events in energy bin i

$\tilde{y}_i$: predicted events, including signal and background appearance and disappearance channels, with neutrino and antineutrino contributions

Statistical treatment

Signal systematic: 5%

Background systematic: 10%

Marginalization over relevant oscillation and nuisance parameters

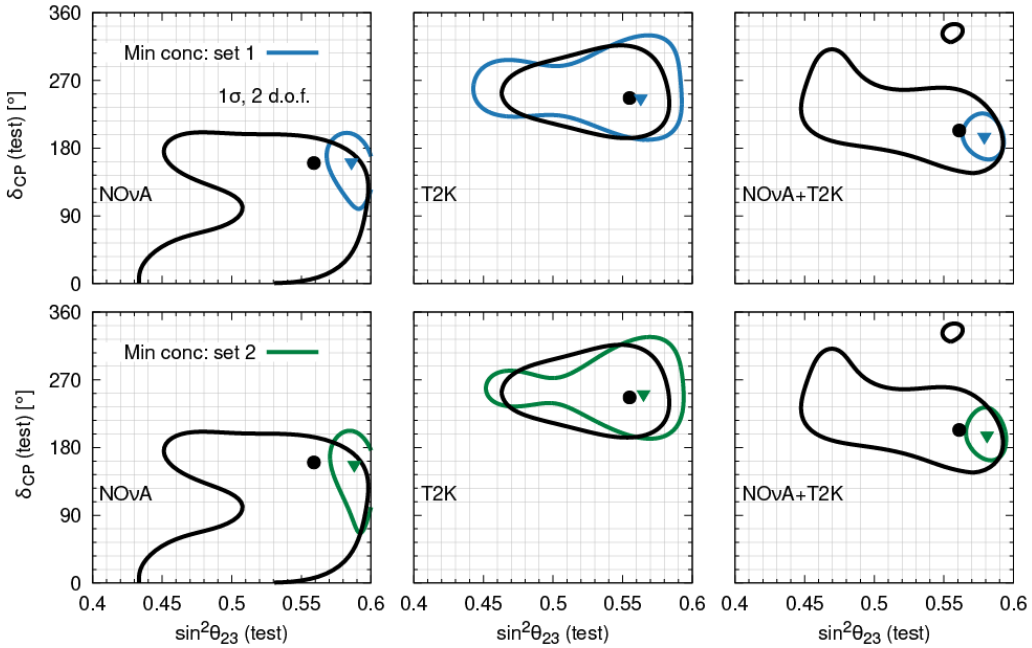
Joint sensitivity $\Delta\chi^2 = \chi^2_{\text{test}} - \chi^2_{\text{min}}$

Combining the T2K and NOνA likelihood contributions to examine allowed regions and parameter sensitivities.

- Entanglement and QFI do not replace the likelihood analysis. They guide our choice of benchmark configurations, while the chi-square quantifies their implications for experimental sensitivity.

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Effect of LMS Strategy on Allowed Regions



Combined fit
NOvA+T2K:
Set 1:

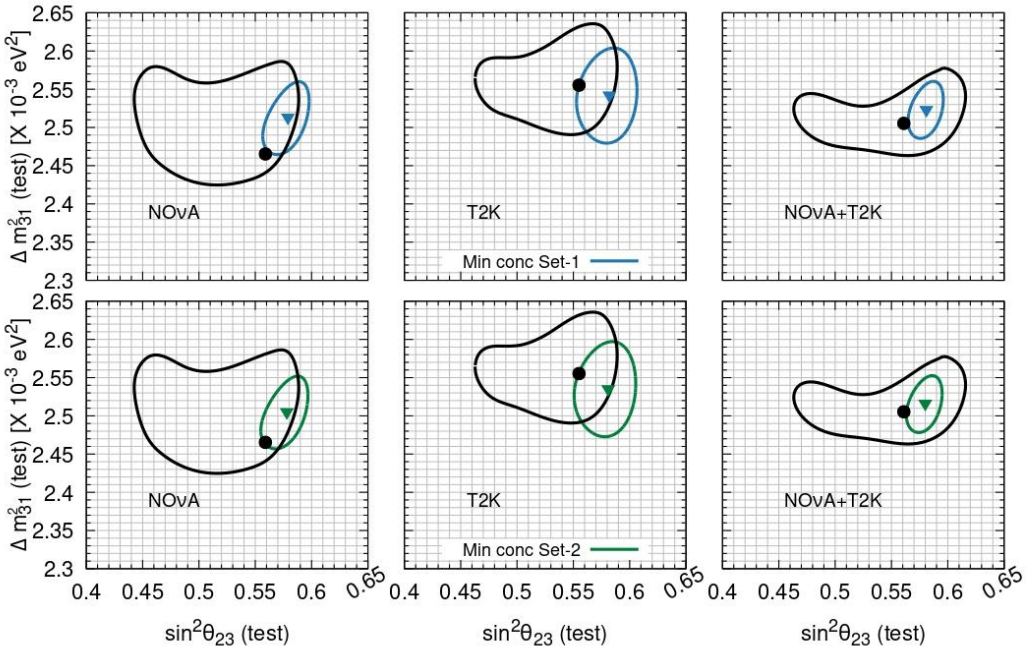
$$(0.579^{+0.0139}_{-0.0154},\ 194^{+32}_{-29}\ ^\circ)$$

Set 2:

$$(0.581^{+0.0136}_{-0.0150},\ 195^{+38}_{-32}\ ^\circ)$$

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Effect of LMS Strategy on Allowed Regions



Combined fit
NOvA+T2K:
Set 1:

$$(0.581^{+0.0140}_{-0.0150},\ 2.522^{+0.0364}_{-0.0348} \times 10^{-3}\,\mathrm{eV}^2)$$

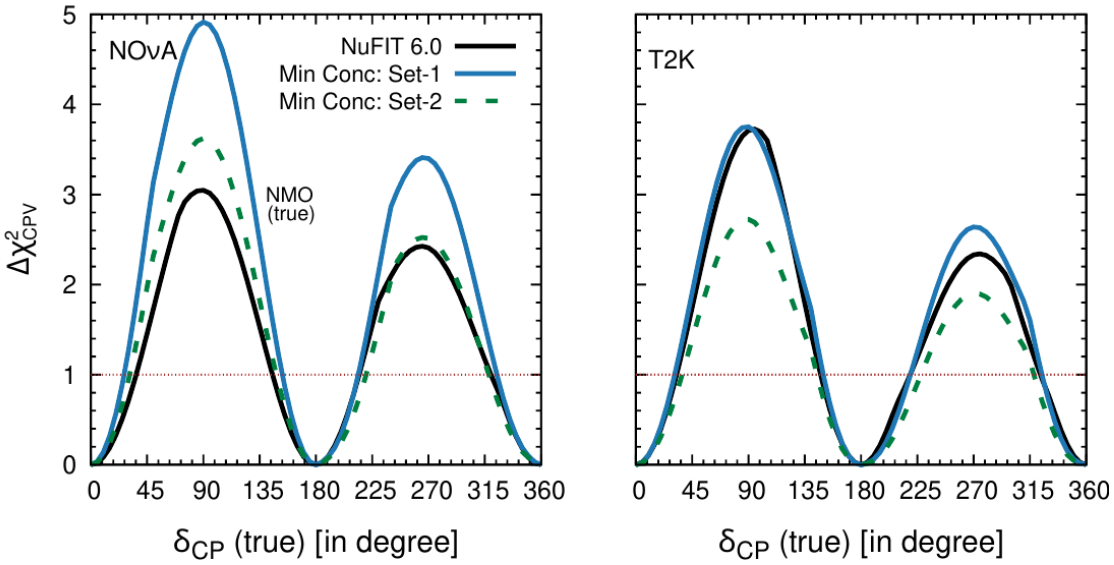
Set 2:

$$(0.580^{+0.0140}_{-0.0153},\ 2.515^{+0.0344}_{-0.0344} \times 10^{-3}\,\mathrm{eV}^2)$$

θ_{23}-octant degeneracy lifted.

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Precision Can Improve, CPV Sensitivity Can Move Both Ways



The strategy is not guaranteed to improve every discovery metric; it mainly improves local parameter precision.

Global sensitivities such as CP violation still depend on full-spectrum distinguishability.

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Overall Takeaways

Neutrino oscillations are not only flavor-transition probabilities, they are coherent quantum-state evolution.

- **Entanglement/concurrence identifies "clean" parameter-sensitive regions:** local minima of total concurrence correspond to nearly separable flavor states, where oscillation parameters can be extracted more clearly.
- **QFI connects QI diagnostics to precision:** concurrence minima occur close to QFI maxima, where the quantum state carries high intrinsic sensitivity to parameters such as δ_{CP}.
- **Practical message for NOνA + T2K:** shifting concurrence minima toward high-flux energy regions improved statistical weight and could tighten joint constraints on oscillation parameters such as θ_{23}, δ_{CP}, and Δm^2_{31}.
- QI tools can potentially guide future experimental designs for better precision and information extraction from particle states.


Thank
you!